Initial Conditions for Early Universe Scenarios

Robert Brandenberger, McGill University

September 21, 2009

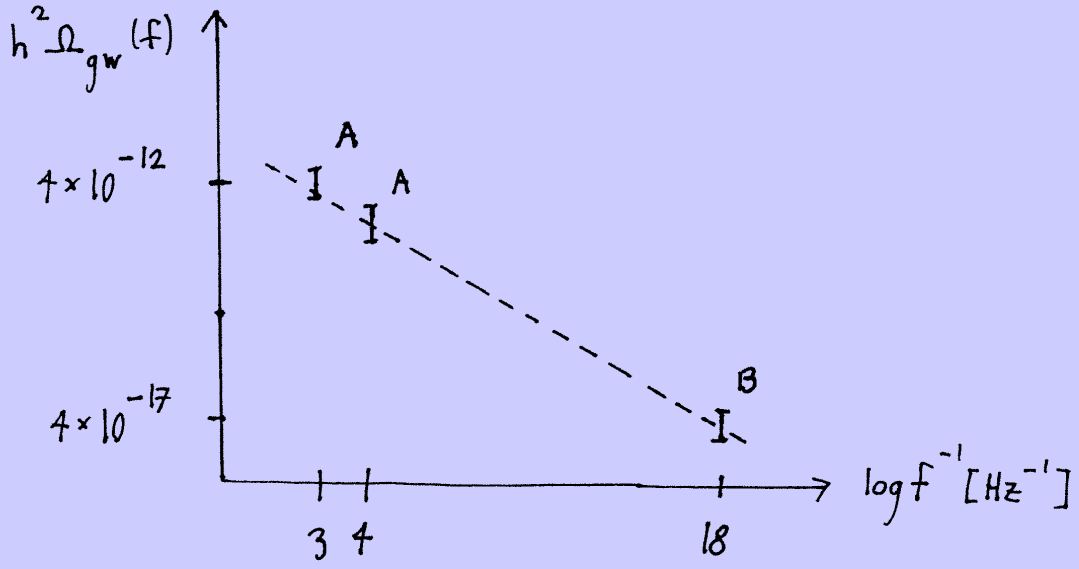
Outline

Plan

test

561|3 September 2020 nature

NEWS & VIEWS



A LISA Collaboration
Phys. Rev. D102 : 043516, 2020

B CBPol Collaboration
Phys. Rev. D96 : 031204, 2017

Inflationary Universe Scenario

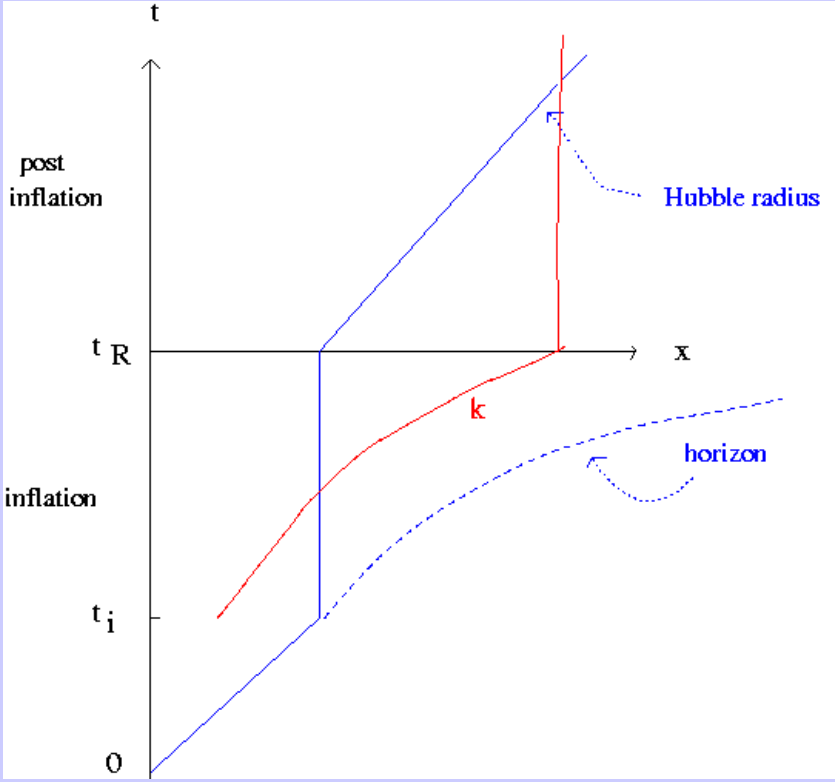
The Inflationary Universe Scenario is the current paradigm of early universe cosmology.

Time line of inflationary cosmology:

- t_i: inflation begins
- t_R: inflation ends, reheating

Space-Time Sketch of Inflationary Cosmology

Space-time sketch of inflationary cosmology:



Note:

- Hubble radius $\equiv H^{-1}$ where $H = \frac{\dot{a}}{a}$
- curve labelled by k: wavelength of a fluctuation

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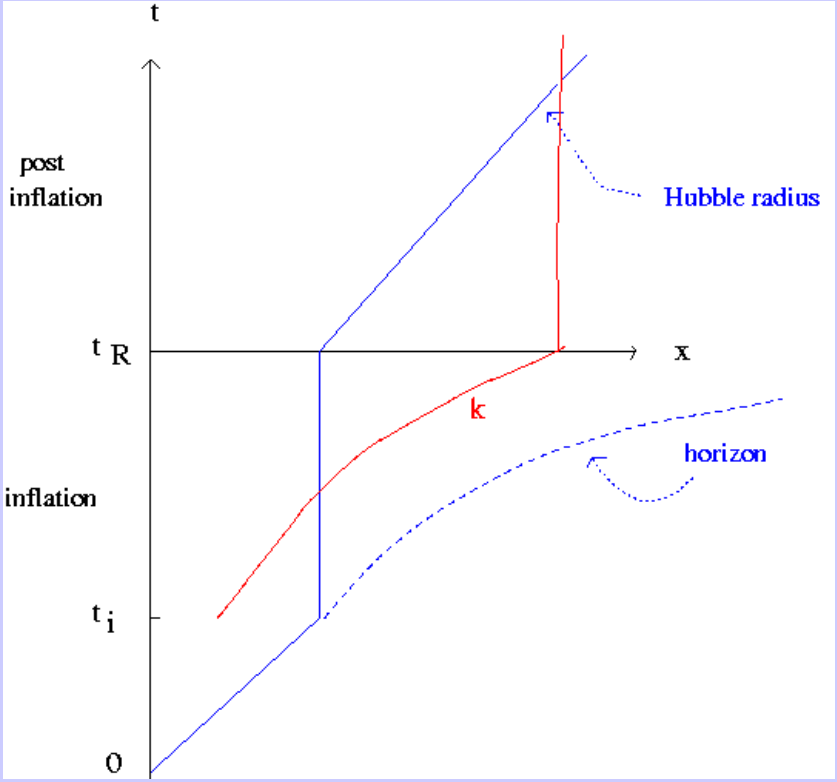

Note:

- Hubble radius $\equiv H^{-1}$ where $H = \frac{\dot{a}}{a}$
- curve labelled by k: wavelength of a fluctuation

Successes of Inflationary Cosmology

Inflationary cosmology:

- Solves horizon problem
- Solves flatness problem
- Solves size/entropy problem
- Provides a causal mechanism of generating primordial cosmological perturbations (Chibisov & Mukhanov, 1981).

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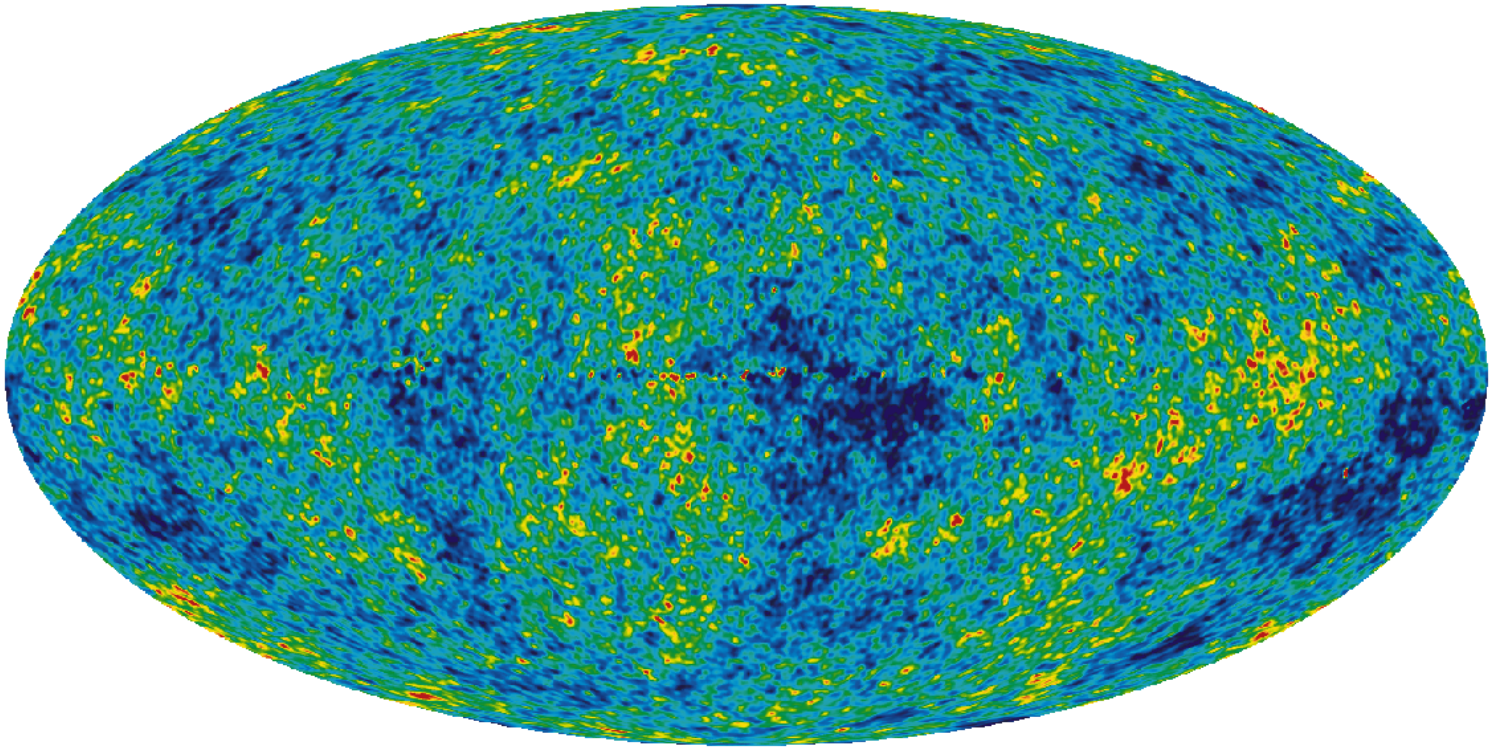
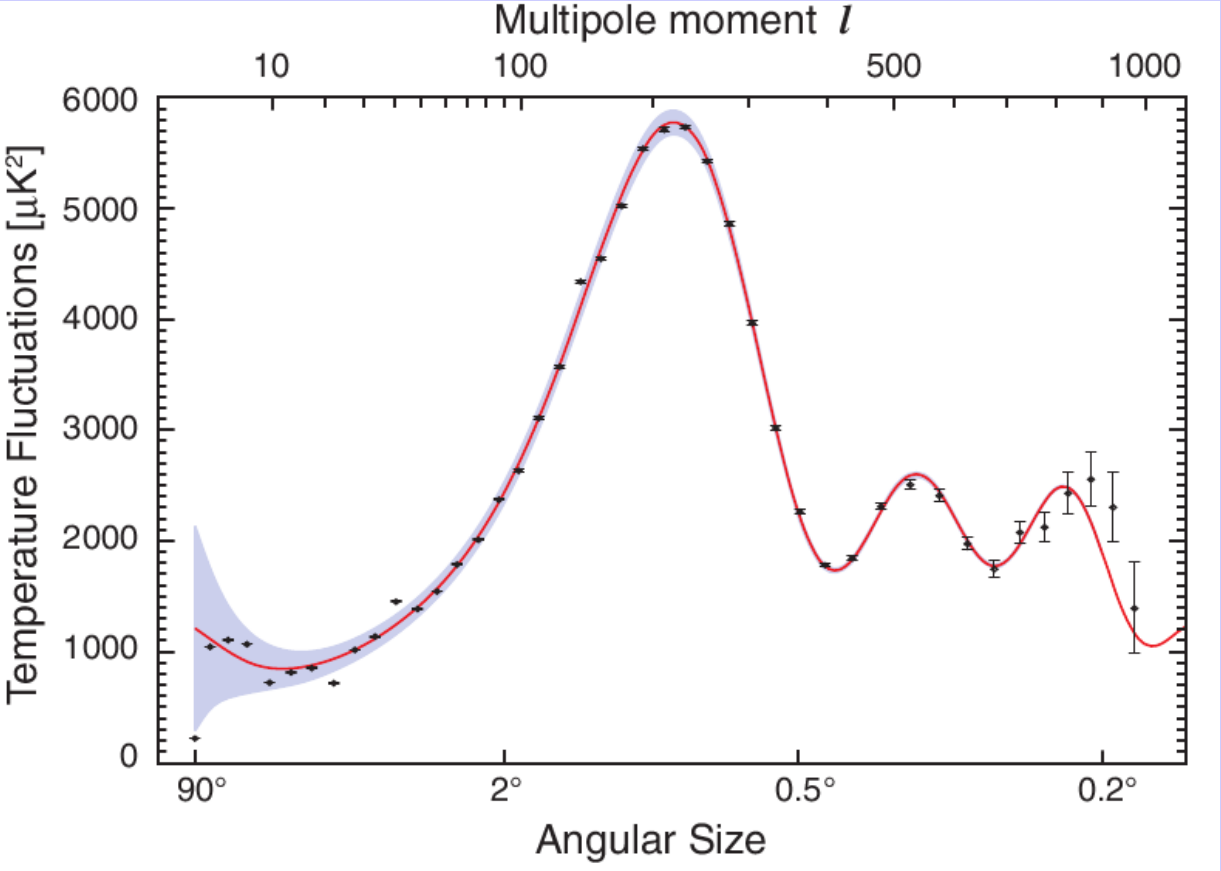


Credit: NASA/WMAP Science Team



Credit: NASA/WMAP Science Team

What do the successes of inflation depend on?

The successes of inflation rely on:

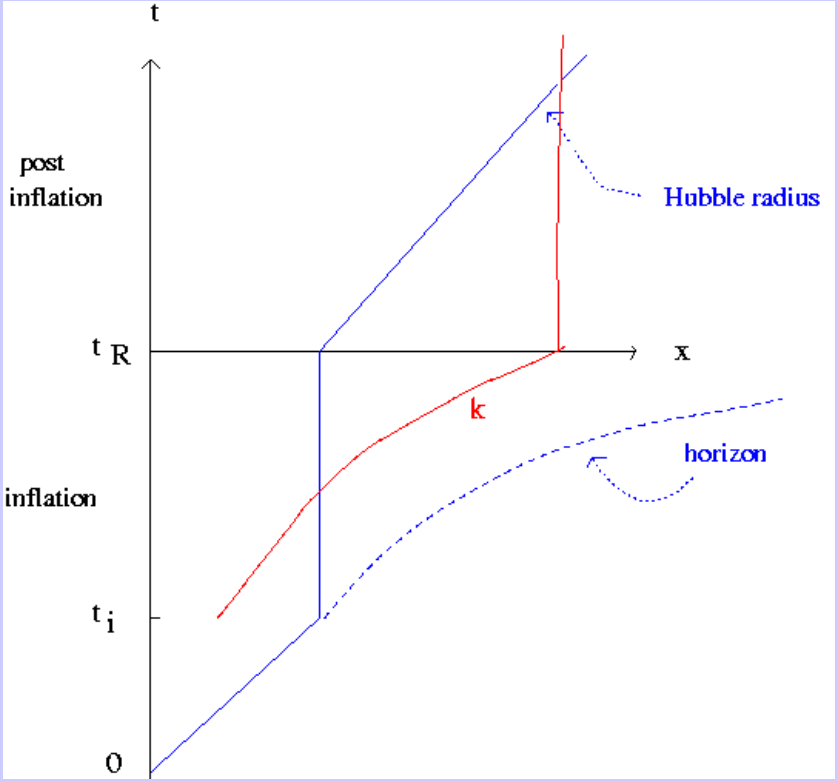
- Mechanism of producing a scale-invariant spectrum of primordial curvature fluctuations on sub-Hubble scales.
- “Free" propagation of fluctuations on super-Hubble scales [the technical term is squeezing].

Hubble radius versus Horizon

- Hubble radius $\equiv H^{-1}$.
- Horizon: extent of the forward light cone.

- Hubble radius separates scales where microphysics dominates (sub-Hubble) from scales where microphysics is frozen out (super-Hubble). Microphysics can only create perturbations on sub-Hubble scales.
- Horizon gives region of causal contact.

Note: In all cosmologies except in Standard Big Bang cosmology the Hubble radius and the horizon are dramatically different!


t
post
inflation
Hubble radius
t R
x
k
horizon
inflation
t i
0

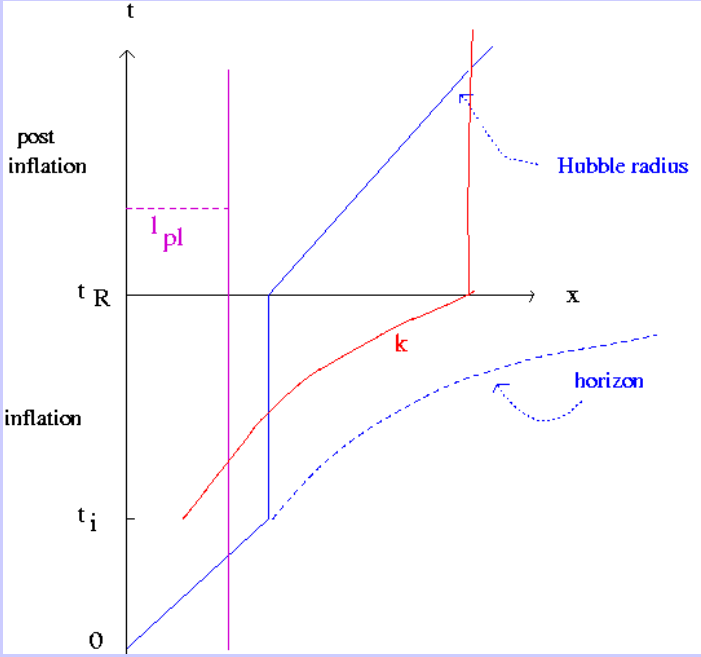
Alternatives: A Preview

- Inflation is not the only way to obtain fluctuations consistent with the current data.
- Studying predictions of alternative models will help sharpen the future predictive power of inflation.
- Alternative models may provide solutions to some of the conceptual problems of current realizations of inflation.

Challenges for the Current Paradigm

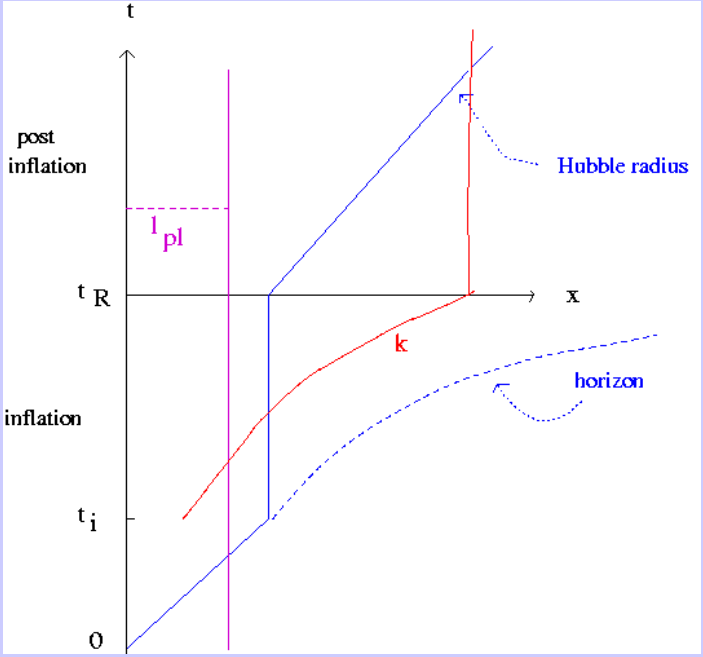
- Nature of the scalar field φ (the “inflaton")
- Conditions to obtain inflation (initial conditions, slow-roll conditions, graceful exit and reheating, amplitude of fluctuations)
- Trans-Planckian problem
- Singularity problem
- Cosmological constant problem
- Applicability of General Relativity

Trans-Planckian Problem



- **Success of inflation**: At early times scales are inside the Hubble radius $\rightarrow$ causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation
- $\rightarrow$ new physics MUST enter into the calculation of the fluctuations.

Trans-Planckian Problem



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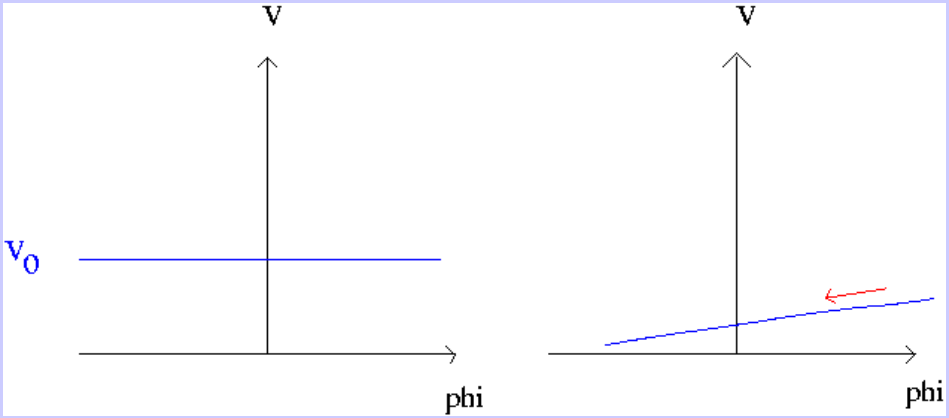
Cosmological Constant Problem



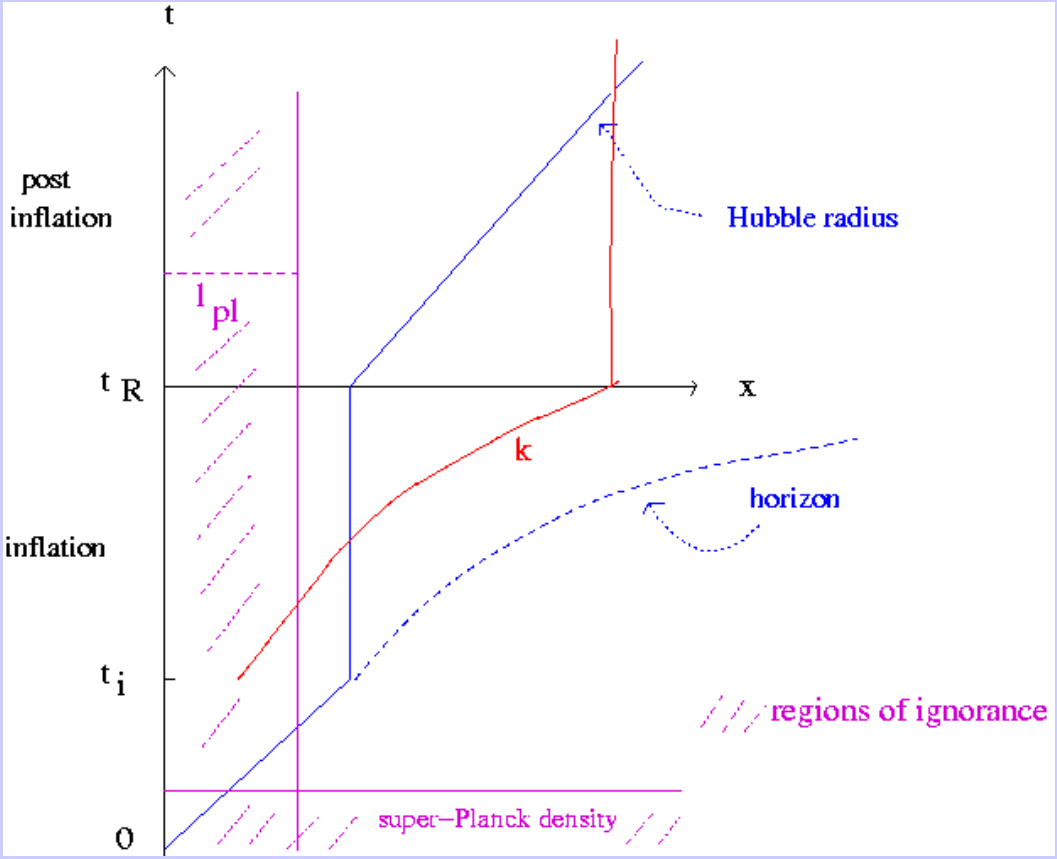
- Quantum vacuum energy does not gravitate.
- Why should the almost constant $V(\varphi)$ gravitate?

$$\frac{V_0}{\Lambda_{obs}} \sim 10^{120} \tag{1}$$

Applicability of GR

- In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion.
- Correction terms may become dominant at much lower energies than the Planck scale.
- Correction terms will dominate the dynamics at high curvatures.
- The energy scale of inflation models is typically $\eta \sim 10^{16}$GeV.
- $\rightarrow \eta$ too close to m_{pl} to trust predictions made using GR.

Zones of Ignorance

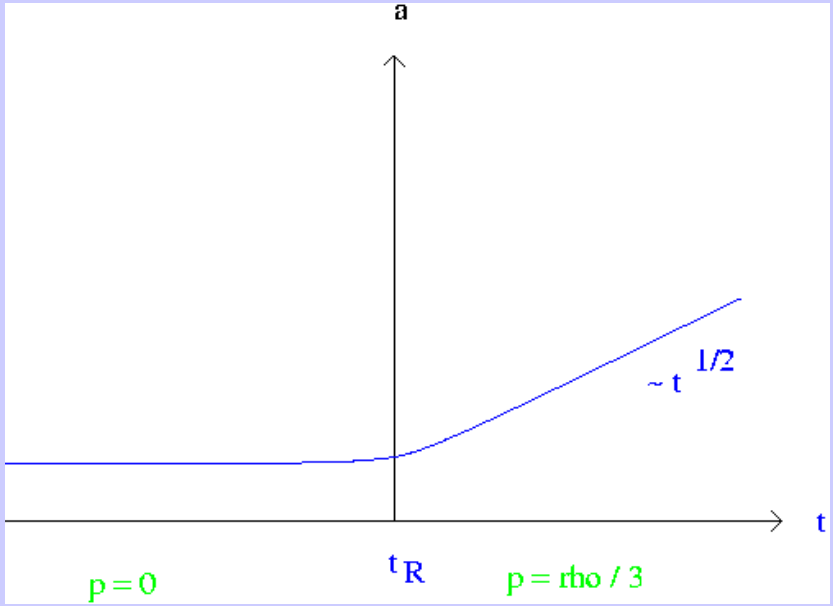


Plan

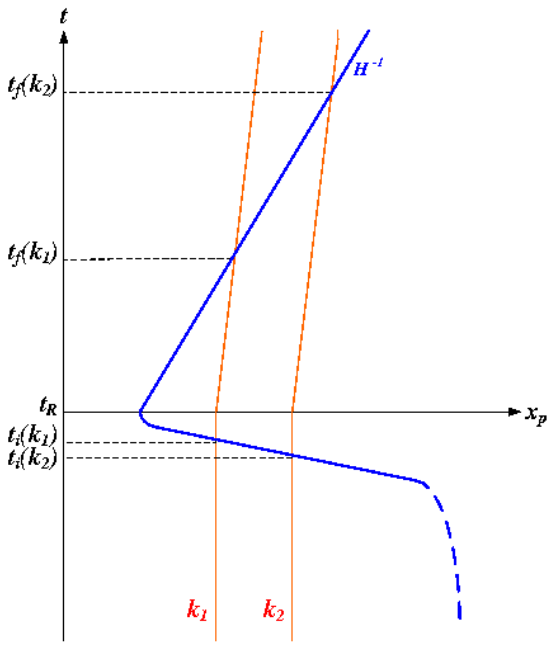
Alternative 1: String Gas Cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett.* **97**, *021302 (2006)*

Idea: Early quasi-static **Hagedorn phase** of strings transiting smoothly to the radiation phase of standard cosmology.



Space-Time Sketch of String Gas Cosmology



Predictions of SGC

Predictions of SGC:

- Scale-invariant spectrum of cosmological perturbations
- Slight red tilt of the spectrum
- Scale-invariant spectrum of gravitational waves
- Slight BLUE TILT of the spectrum (unlike inflation!) [R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett.* **98**, *231302 (2007)*]

Note: No trans-Planckian problem for fluctuations (wavelength always in the far IR).

Principles of String Gas Cosmology

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

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T-Duality

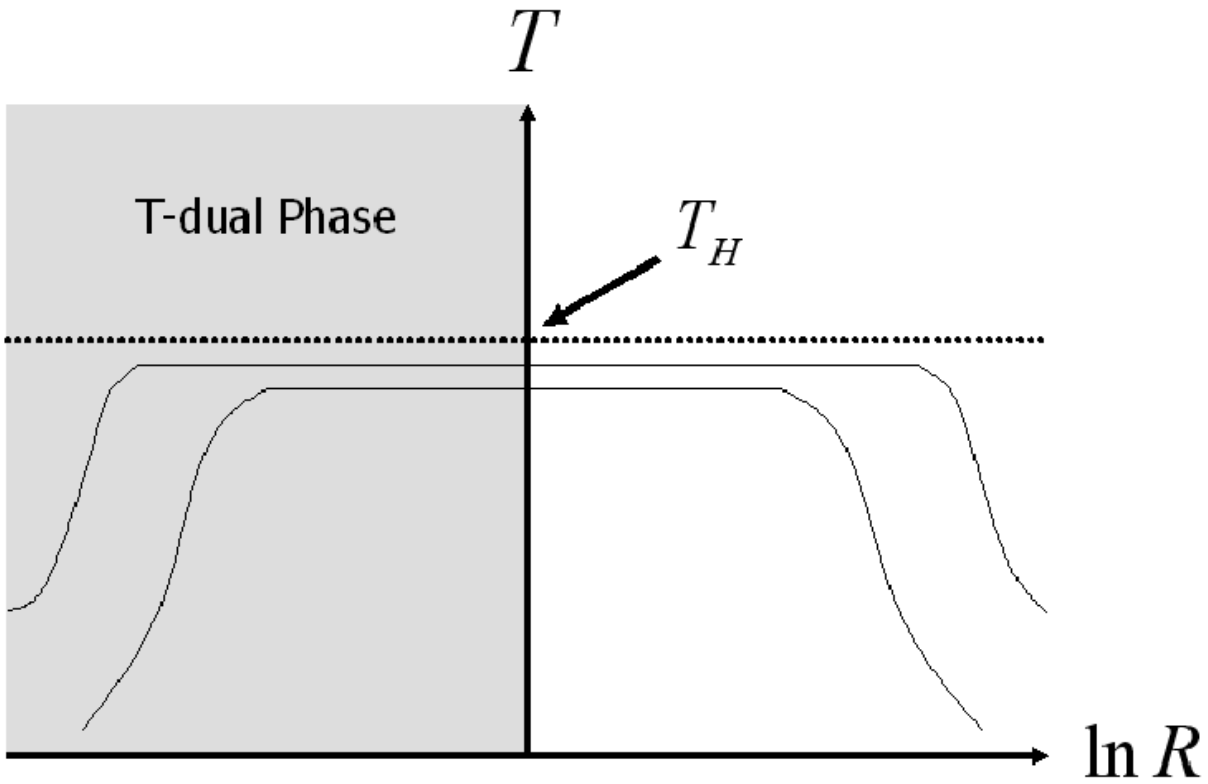
T-Duality

- Momentum modes: $E_n = R/n$
- Winding modes: $E_m = mR$
- Duality: $R \to 1/R \quad (n, m) \to (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\to$ existence of D-branes

Adiabatic Considerations

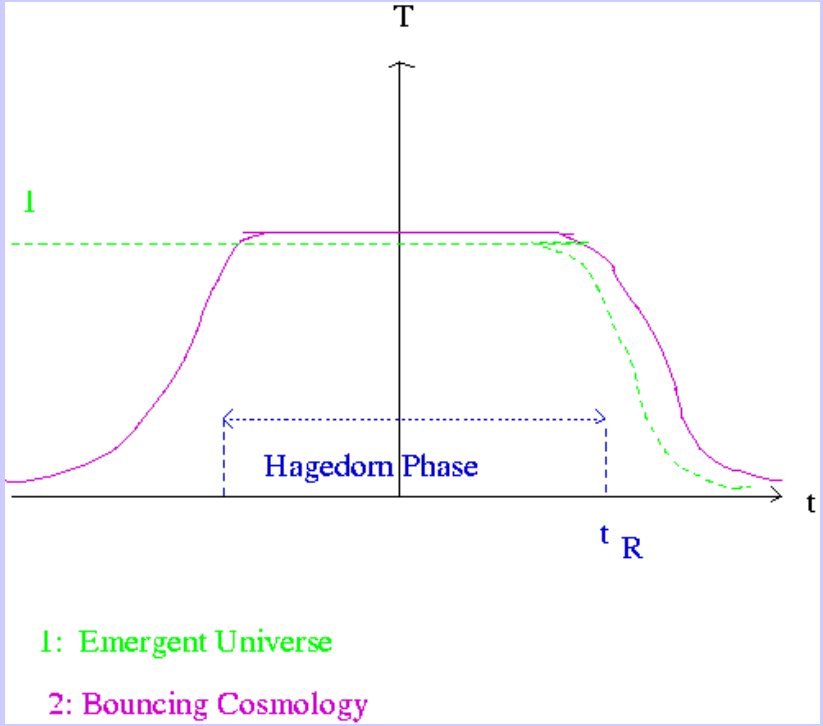
R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



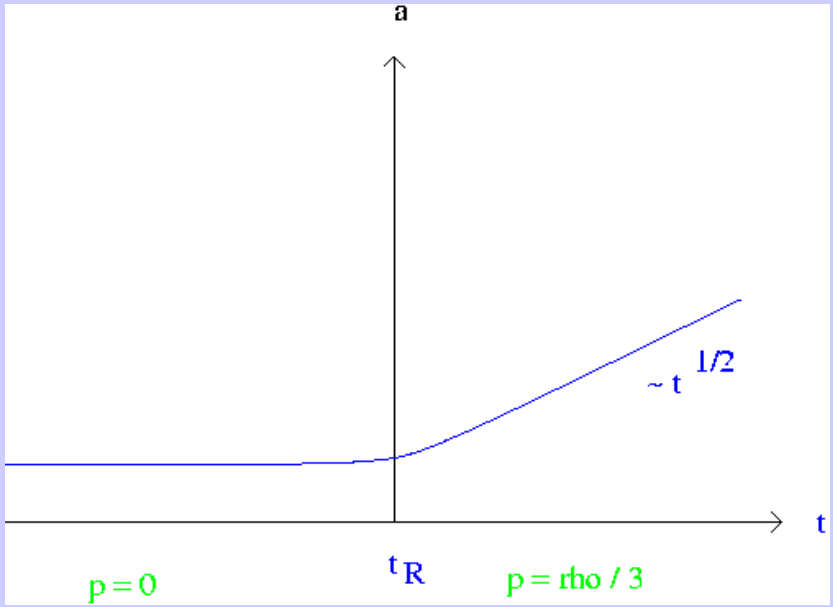
Dynamics

Assume some action gives us $R(t)$



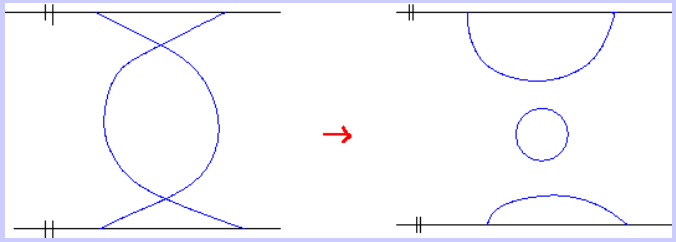
Dynamics II

We will thus consider the following background dynamics for the scale factor $a(t)$:



Dimensionality of Space in SGC

- Begin with all 9 spatial dimensions small, initial temperature close to T_H $\rightarrow$ winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.



- Decay only possible in three large spatial dimensions.
- $\rightarrow$ dynamical explanation of why there are exactly three large spatial dimensions.

Note: this argument assumes constant dilaton [R. Danos, A. Frey and A. Mazumdar]

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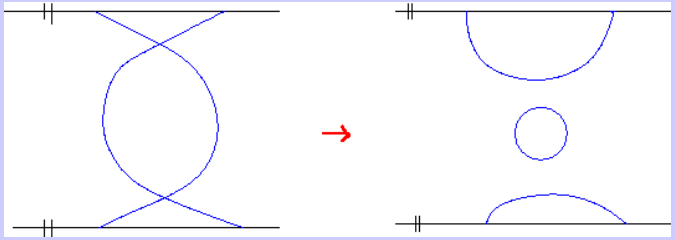

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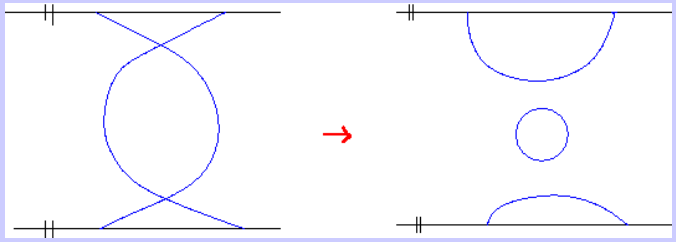

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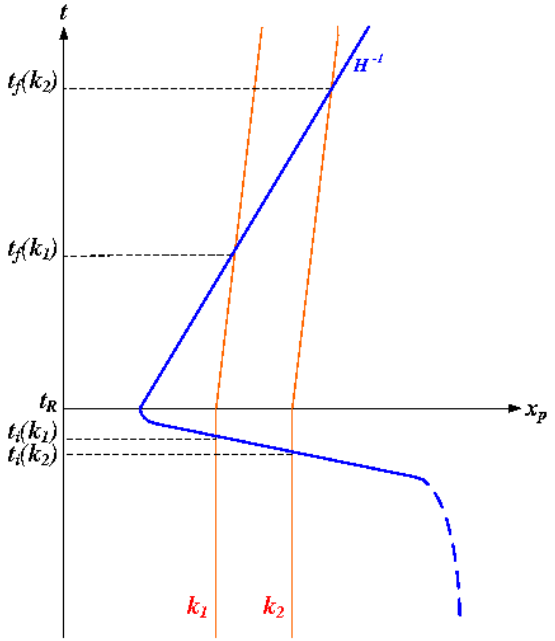
Origin of Fluctuations in String Gas Cosmology

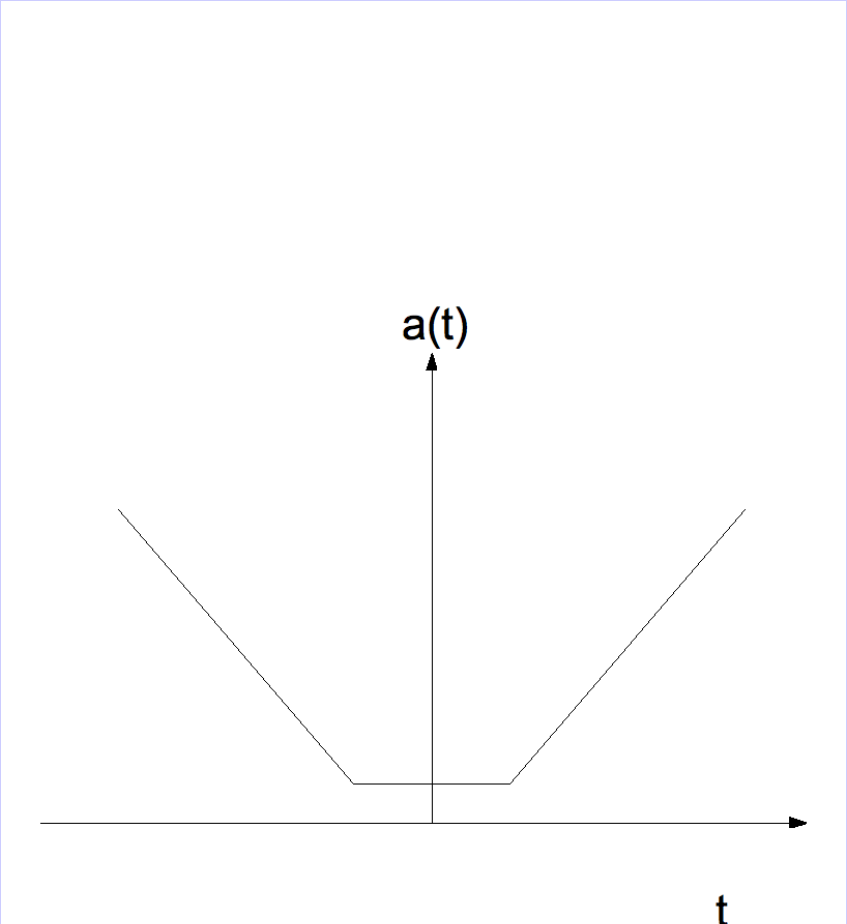
- Early phase dominated by a hot gas of fundamental strings $\rightarrow$ fluctuations originate from thermal fluctuations of a string gas.
- Crucial to obtain a scale-invariant spectrum of curvature fluctuations at late times is the holographic scaling of string thermal fluctuations [specific heat capacity $C_V(R) \sim R^2$ determines the density fluctuations].
- Pressure deeper in the Hagedorn phase is closer to zero $\rightarrow$ lower amplitude of gravitational waves on large scales. $\rightarrow$ blue spectrum of gravitational waves.


t
$t_f(k_2)$
$t_f(k_1)$
t_R
$t_i(k_1)$
$t_i(k_2)$
H^{-1}
x_p
k_1
k_2

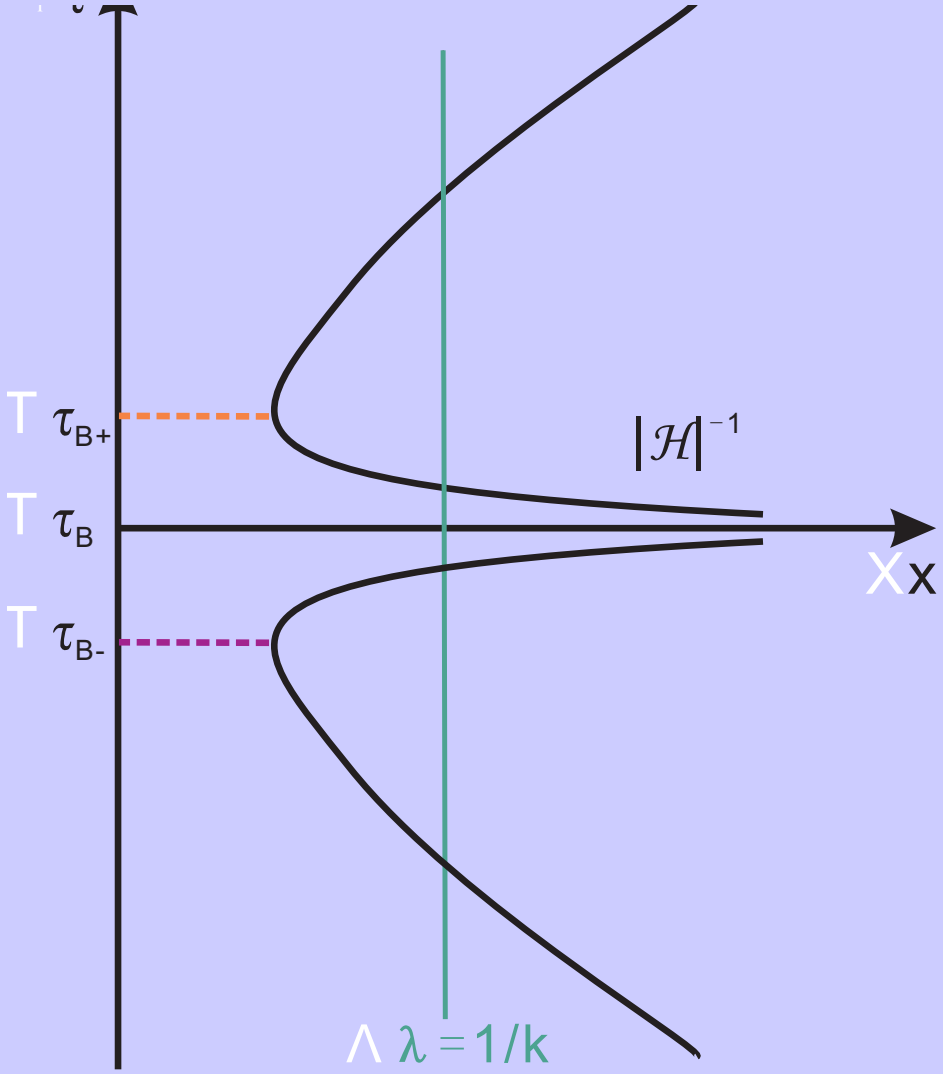
Alternative 2: Matter Bounce Cosmology

D. Wands, Phys. Rev. D **60**, 023507 (1999): F. Finelli and R.B. Phys. Rev. D **65**, 103522 (2002); Y. Cai, T. Qiu, R.B. and X. Zhang, arXiv:0810.4677

Idea: Non-singular bouncing cosmology with a matter-dominated phase of contraction, can be realized in the context of Horava-Lifshitz gravity [R.B., arXiv:0904.2835].



Space-Time Sketch of Matter Bounce Cosmology



Predictions of Matter Bounce Cosmology

Predictions of Matter Bounce Cosmology:

- Non-singular cosmological background
- Scale-invariant spectrum of cosmological perturbations
- Scale-invariant spectrum of gravitational waves
- Unsuppressed tensor to scalar ratio
- Large amplitude and specific shape of the bispectrum [Y. Cai, W. Xue, R.B. and X. Zhang, JCAP **0905**:011 (2009)].

Note: No trans-Planckian problem for fluctuations (wavelength always in the far IR).

Realizing a Matter Bounce

There are a number of ways to realize a matter bounce, none of them at the present time entirely satisfactory:

- Higher derivative gravity model of Biswas et al. constructed to be ghost-free about Minkowski space-time [T. Biswas, A. Mazumdar and W. Siegel, JCAP **0603**, 009 (2006)].
- Horava-Lifshitz gravity: A power-counting renormalizable theory of gravity based on anisotropic scaling of space and time. Higher spatial derivative terms are added to the action. They render the theory better behaved in the UV. Coupled to spatial curvature, they allow a non-singular bouncing cosmology
- quintom matter: An effective field theory which at low energies contains a field with negative kinetic term: [Y.Cai et al, JCAP **0803**, 013 (2008)]

Origin of Fluctuations in the Matter Bounce

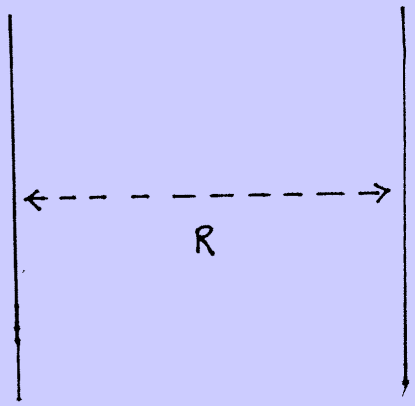
It is assumed that fluctuations emerge as quantum vacuum perturbations on sub-Hubble scale, as in the case of inflation.

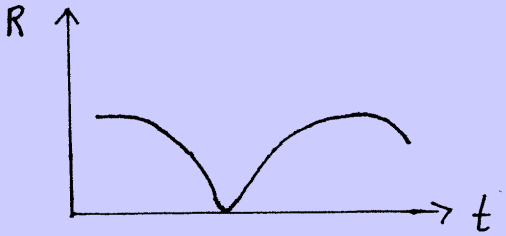
Unlike in the case of inflation, there is no good reason for taking these intitial conditions (see later).

Alternative 3: Ekpyrotic/Cyclic Scenario

J. Khoury et al, Phys. Rev. D **64**, 123522 (2001)

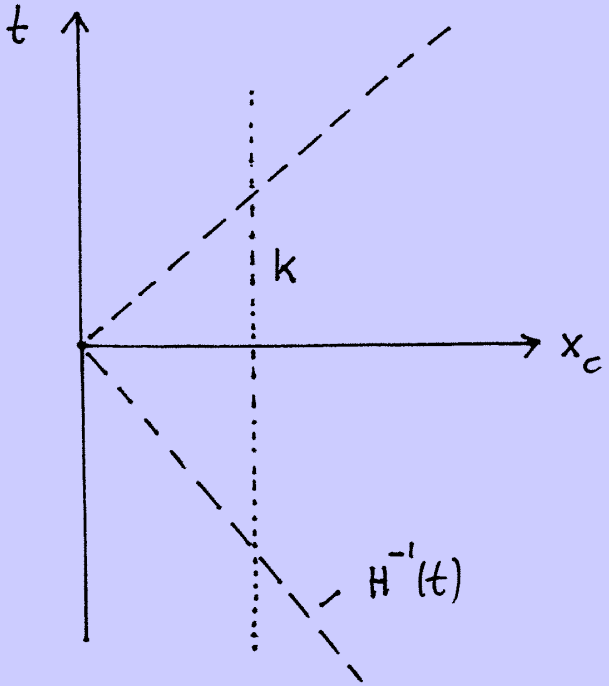
Ekpyrotic/Cyclic cosmology emerges from a string theory-inpired framework in which the radius of an extra dimension bounces/cycles.





Space-time sketch

The space-time sketch of the effective 4-d theory which determines cosmological fluctuations is:



Predictions

- In the absence of a resolution of the singularity at the bounce point it is dangerous to make predictions.
- Provided that the pre-bounce growing mode of the fluctuations couples non-trivially to the dominant mode in the post-bounce phase, then a scale-invariant spectrum of cosmological perturbations results.
- Resolving the 5-d bounce using a gas of strings wrapping the 5th dimension yields a non-vanishing coupling [T. Battefeld, S. Patil and R.B., Phys. Rev. **D73**:086002 (2006)].
- This confirms the analytical continuation argument by Tolley and Turok [A. Tolley and N. Turok., Phys. Rev. **D69**:106005 (2004)].

Origin of Fluctuations in the Ekpyrotic Scenario

It is assumed that fluctuations emerge as quantum vacuum perturbations on sub-Hubble scale, as in the case of inflation.

In the cyclic version of the scenario, these intitial conditions are justified by the red-shifting of pre-bounce fluctuations in the exponential expansion phase before contraction (the same mechanism as in the inflationary scenario.

Plan

Spatial Curvature Problem

- Inflation starts at energy scales $E \sim 10^{16}\mathrm{GeV}$, several orders of magnitude smaller than the Planck scale.
- Pre-inflationary dynamics must allow the universe to cool sufficiently.
- Initial spatial curvature must be sufficiently small in order that the universe not re-collapse before then.

Small Field Inflation

A. Linde, 1982; A. Albrecht and P. Steinhardt, 1982.

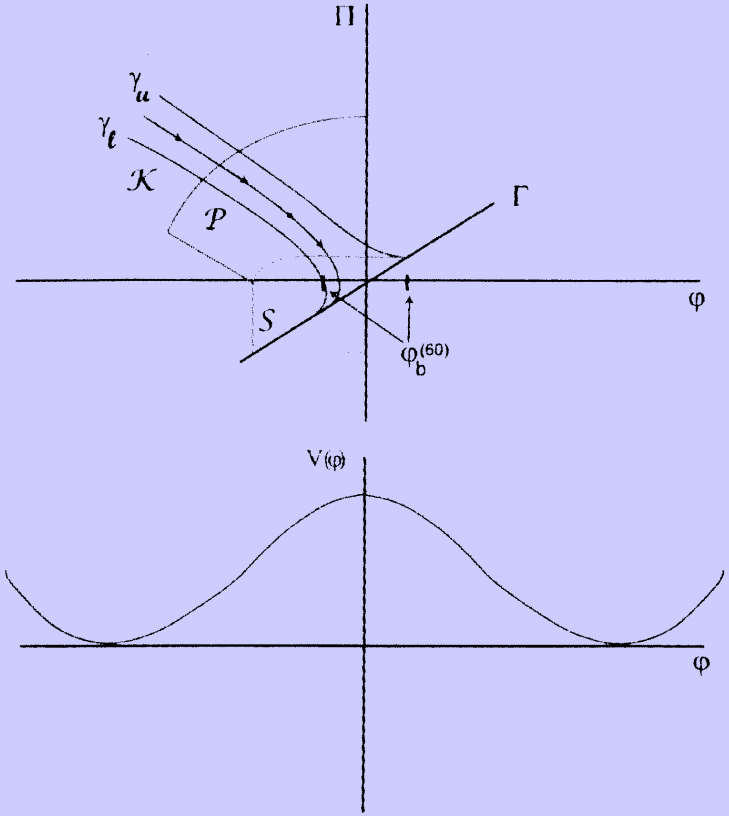
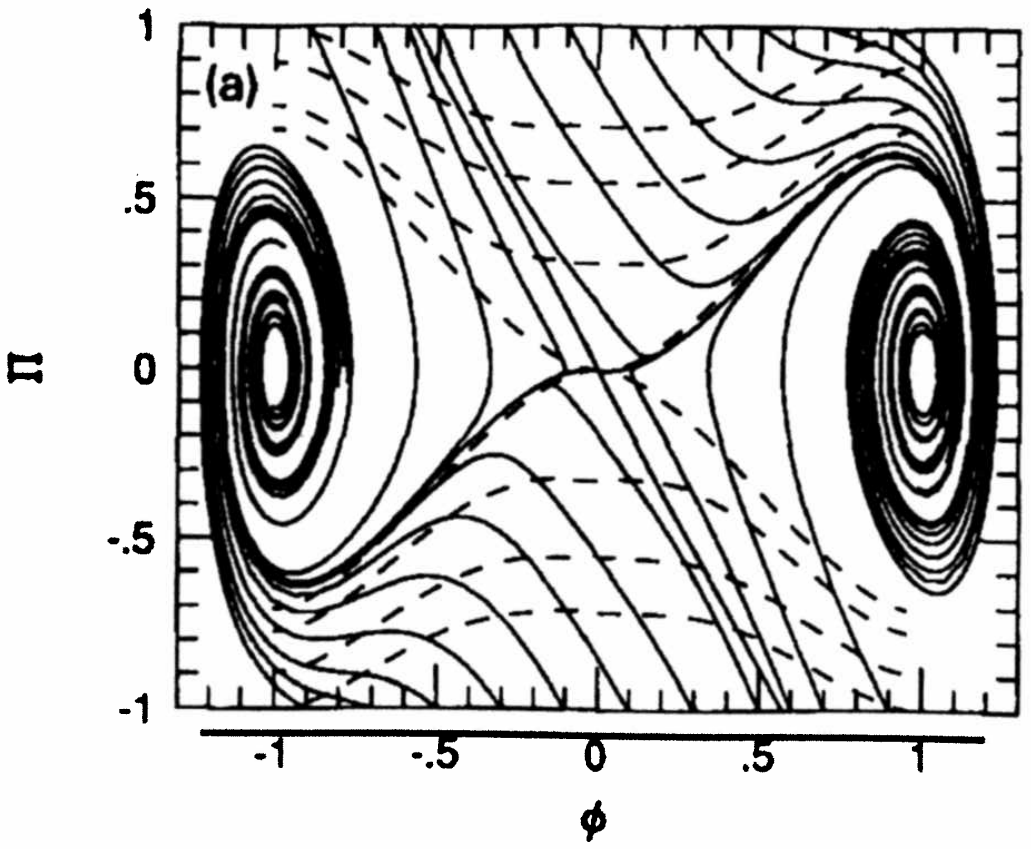


FIG. 1. Sketch of the potential (lower figure) and phase space in a model of small field inflation. The region which leads to successful slow roll inflation lies between the the lower and upper diagonal curves γ_l and γ_u in the upper left quadrant. The symbols are defined in the text.

Dynamics in Small Field Inflation

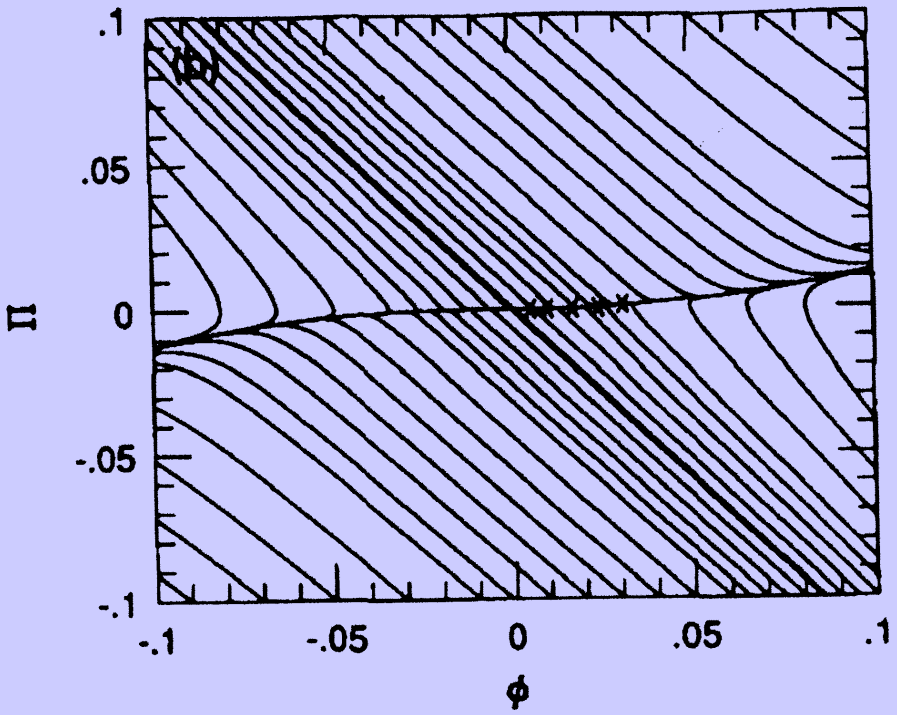
D. Goldwirth and T. Piran, Phys. Rept. **214**, 223 (1992).



Initial Condition Problem for Small Field Inflation

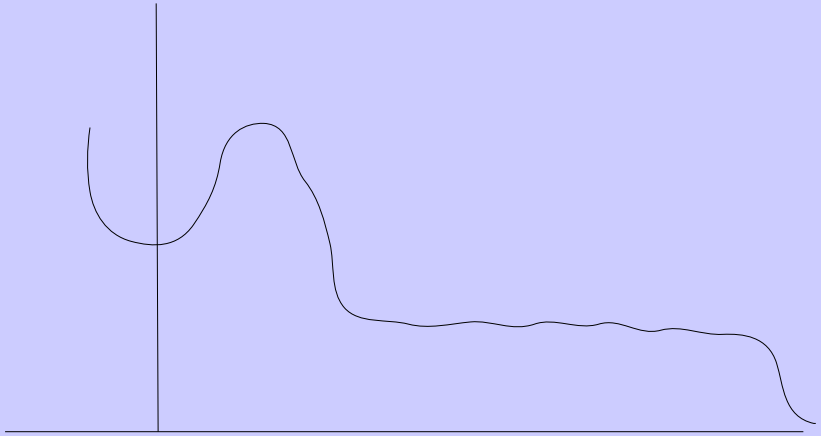
D. Goldwirth and T. Piran, Phys. Rept. **214**, 223 (1992).

Trajectories yielding sufficient inflation are marked with x. Their phase space is very small.

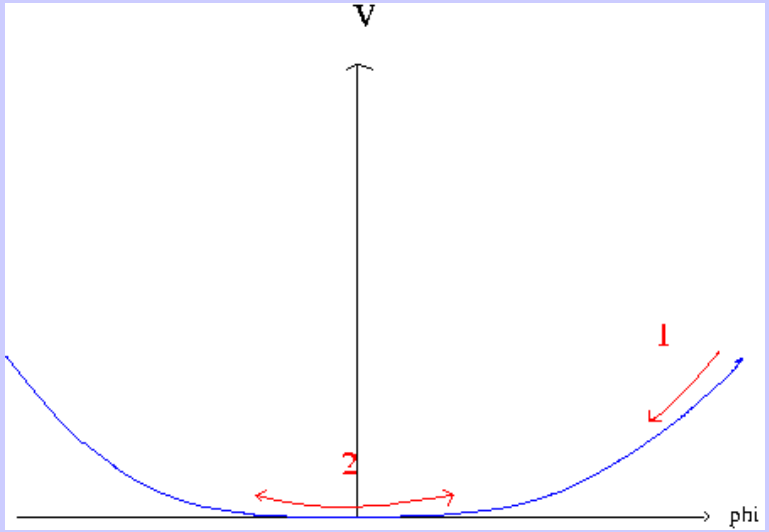


Resolution of the Problem

Initial conditions are realized without fine-tuning if slow-roll inflation begins after tunneling from a false vacuum.



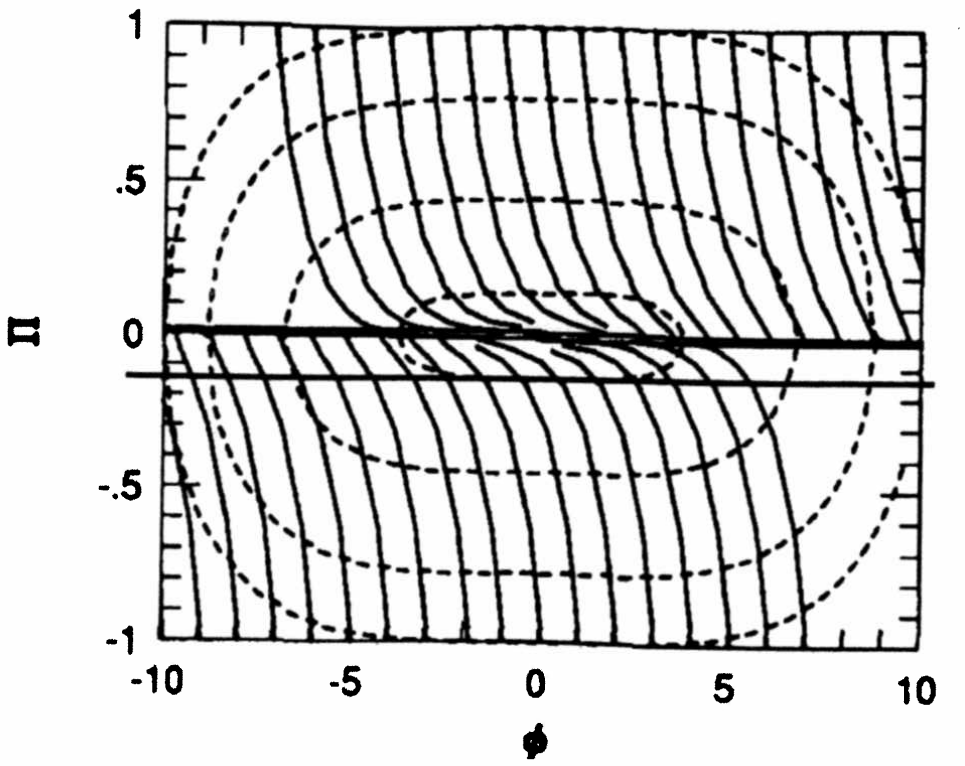
Large Field Inflation
A. Linde, 1983



Note: The region of inflation corresponds to field values larger than m_{pl}!

Slow Roll Trajectory as a Local Attractor in Initial Condition Space

D.S. Goldwirth and T. Piran, Initial conditions for inflation



Slow Roll Trajectory as a Local Attractor in Initial Condition Space II

J. Kung and R.B., Phys. Rev. **D42**, 1008 (1990).

The slow roll trajectory remains an attractor when introducing inhomogeneities.

Note: The initial energy density does not have to be dominated by the homogeneous mode!

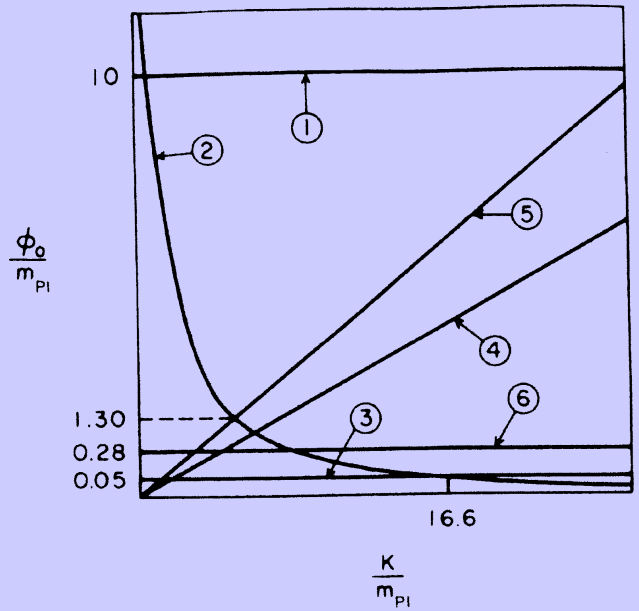


FIG. 3. The single-Fourier-mode initial-condition space (static case). Chaotic inflation occurs for points above line 6, to the left of line 5, below line 1, and to the left of line 2. The latter two conditions come from *a priori* energy density constraints.

Slow Roll Trajectory as a Local Attractor in Initial Condition Space III

H. Feldman and R.B., Phys. Lett. **227B**, 359 (1989).

Note: Linear gravitational inhomogeneities can be introduced without changing the local attractor nature of the slow-roll trajectory.

Note: In the case of hybrid inflation the initial condition issue is like for large-field inflation.

Initial Conditions for String Gas Cosmology

- Thermal equilibrium of the string gas is a local attractor in initial condition space.
- Matter is relativistic $\rightarrow$ Jeans length comparable to the Hubble length.
- Homogenization on scales smaller than the Jeans length.
- Assuming a quasi-static Hagedorn phase which is long-lived, the homogenization mechanism acts on scales sufficient to explain the current overall homogeneity.

Initial Conditions for Matter Bounce
(homogeneous level)

- A matter dominated phase of contraction is unstable towards radiation, anisotropic stress, ...
- $\rightarrow$ even at the level of background cosmology, a matter bounce generated by matter effects is highly unlikely.
- If the cosmological bounce is induced by extra terms in the gravitational action whose effective energy density scales as a^{-6}, then the bounce is stable towards perturbations in the initial conditions (at the level of homogeneous cosmology).
- This is the case in Horava-Lifshitz gravity.
- The initial universe needs to be very large [N. Kaloper, A. Linde and R. Bousso, Phys. Rev. **D59**:043508 (1999)] - but is this a problem in the context of a cold initial universe?

Initial Conditions for Matter Bounce
(inhomogeneous level)

- In a pure matter bounce (matter-dominated until the bounce) a fine-tuning of initial conditions is required in order to prevent a collapse into a gas of black holes on all length scales.
- The finite Jeans length in a radiation phase will not prevent the collapse on super-Hubble scales into black holes.
- Even for more general bounces (matter-dominated only during the initial phase of contraction) fine-tuning of the initial classical fluctuations is required [R. Kallosh, L. Kofman and A. Linde, Phys. Rev. **D64**:123523 (2001)].

Initial Conditions for the Ekpyrotic/Cyclic Scenario

- Equation of state $w > 1$ during the phase of contraction ensures ultralocality and thus supports the existence of the bounce for a wider range of initial conditions.
- Without the cyclicity assumption there is a severe fine-tuning of initial conditions required in order to obtain a scale-invariant spectrum since classical fluctuations are amplified in the contracting phase [R. Kallosh, L. Kofman and A. Linde, Phys. Rev. **D64**:123523 (2001)].
- The phase of accelerated expansion (inflation) between the cycles leads to exponential red-shifting of pre-existing inhomogeneities (like in inflationary cosmology). Thus, the homogeneous Ekpyrotic trajectory is a local attractor in initial condition space.

Plan

Conclusions

- There are alternatives to inflation for explaining the origin of structure in the universe.
- The alternatives give rise to different predictions for future observations.
- My favorite alternative: String Gas Cosmology. It predicts a blue tilt to the spectrum of gravitational waves.
- A good theory should be a local attractor in initial condition space.
- Some alternatives to inflation do not satisfy this condition.
- Large field inflation is a local attractor in initial condition space.
- In the context of the string landscape tunneling picture the initial conditions for small field inflation (which at first sight seem to require fine-tuning) can be realized.

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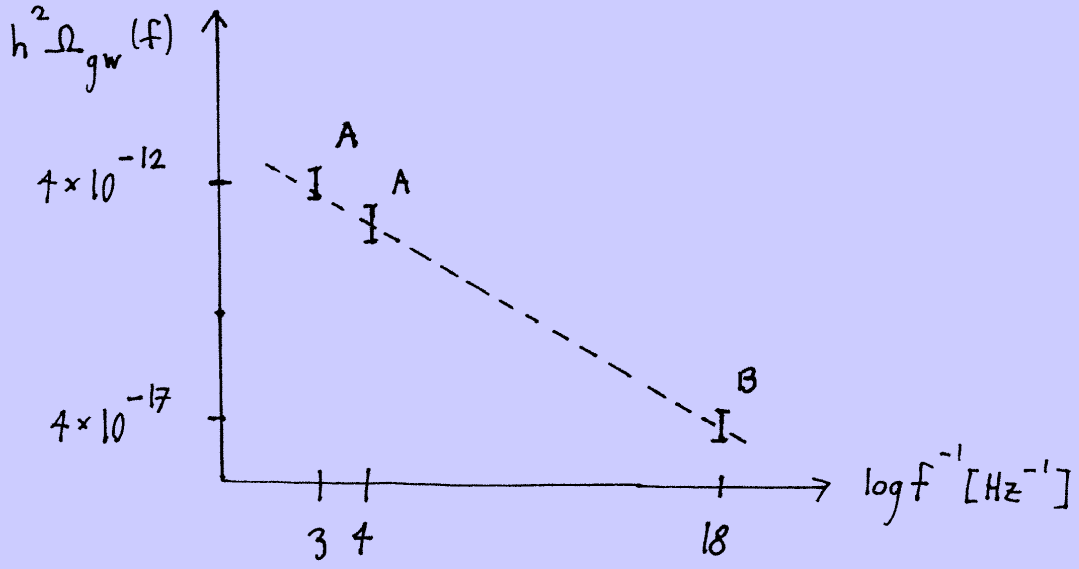
Data will tell!

561|3 September 2020 nature

NEWS & VIEWS



A LISA Collaboration
Phys. Rev. D102 : 043516, 2020

B CBPol Collaboration
Phys. Rev. D96 : 031204, 2017