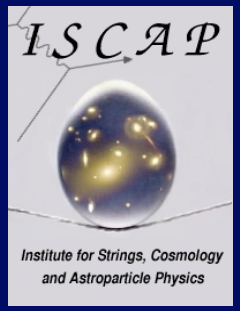
Multiverse Panel: A Few Comments



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Oxford, September 21, 2009.

Science in a Multiverse

- Can a Multiverse proposal be tested?
- No universal answer
- Depends on which kind of multiverse
- Depends on detailed way multiverse is realized.
- In-principle vs in-practice considerations

Science in a Multiverse
Direct Tests?

- Infinite Expanse:
  - Patterns in CMB
- Brane World Models:
  - Missing Energy signatures
  - Black Hole production
- Quantum Mechanical Many Worlds:
  - Collapse theories/Unitary Evolution.
- Bubble Universes:
  - Bubble Collisions.

Science in a Multiverse
General In-Principle Tests

- A Uniform Multiverse
  - Unique predictions across universes.
- Strict Correlations within Universes
  - Unique pairings of known/unknown properties.
- Established theory w/ironclad multiverse byproduct.
  - Multiverse must not compromise experimental/ observational vetting of theory.

Science in a Multiverse

Eternal Inflation

- Inflation:
  - Standard successes; CMB fluctuations.
- Subsequent Realization:
  - Generically eternal; pocket universes.
- Measures and their Discontents:
  - Compromises unambiguous multiverse predictions.
- Compromises observational support for inflation.
  - Can only speak of inflation+measure

Probabilities and Measures

- When is a measure interpretable as a probability?
- When does a measure emerge from a theory?
- Quantum Mechanics:
  Frequency Operator: $F_k(A)$; $A;\Psi_k$

$$\Psi = \sum c_k \Psi_k ; A\Psi_k = a_k \Psi_k$$

$$\Psi \rightarrow \Psi^{\otimes n}$$

$$\lim_{n\rightarrow\infty} F_k(A)\Psi^{\otimes n} = |(\Psi,\Psi_k)|^2 \ \Psi^{\otimes n}$$

Probabilities and Measures

- Similar emergence of measure in other theories?
- Similar level of predictive power? Seems far off at best.
- Multiverse: Shifts the kinds of questions we ask.
  Eliminates a class of pursuits.
  If right, hugely valuable; deep insight.
- Two Perspectives
- Dangerous—Premature
- Unnecessary—Nothing to explain.

What Should we be Surprised by?

A Perspective

- Multiverse—Tremendously interesting; natural development. No threat to science.
- Many roads/many versions.
- Important tool in the arsenal.
- 25 years ago—3 gen CYs.
- Still a real possibility.

STRING/M-THEORY COSMOLOGY

- Grand Questions
  - Resolution of big-bang singularity
  - Emergence of spacetime
  - Link to effective description: inflation/FRW
- Specific Issues
  - Why 4 large spacetime dimensions?
  - Observational consequences of string/M-theory cosmology?

Work Done With:

- Richard Easther
- Dan Kabat
- Mark Jackson
- Stefanos Marnerides

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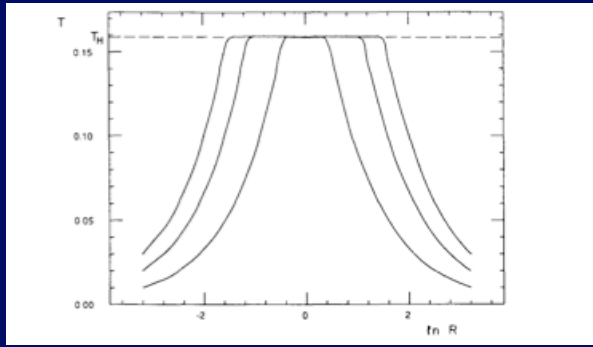
String Gas Cosmology

- Philosophy:
  - Capture: Essential Features of String Theory—Key Differences from Point Particle Theory.
    - New Kinds of States
    - New Symmetries
- Specifically:
    - Winding Modes
    - T-Duality

String Gas Cosmology

- Spatial T-duality→Temperature duality
  - (Brandenberger and Vafa)



String Gas Cosmology

Brandenberger and Vafa (1989)/Tsytelin and Vafa (1991):

- Analyzed thermodynamics of string networks
- Noted that strings only generically interact in three (or fewer) spatial dimensions
  - D-dimensional objects have D+1 dimensional worldvolumes
  - Only interact in 2(D+1) dimensions (or less), in the absence of long range forces
- Considered cosmological consequences of winding strings: $2 + 2 = 3 + 1$

Proposed Cosmological Dynamics

- Winding modes keep spatial section near Planck volume.
- Spatial section thermally fluctuates.
- When 4 or more dimensions fluctuate large, driven back to Planck size.
- When 3 or fewer dimensions fluctuate large, annihilations remove winding modes, continues to expand.

Issues to Consider

- Topology: 2
  - Assumes spacetime is toroidal.
  - Calabi-Yau, Orbifolds, etc.
- Dynamics: 4 1
  - Realize intuitive expecations?
- M-theory: 3
  - Include branes. What happens?
- String Theory 5 6
  - Dilute Gas Regime

1.Dynamics

- Duality Invariant Action:

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g}\, e^{-2\phi} \left(R + 4(\partial\phi)^2 + \cdots \right)$$

$$ds^2 = -dt^2 + \alpha' \sum_{i=1}^{9} e^{2\lambda_i(t)} d\theta_i^2 \qquad 0 \le \theta_i \le 2\pi$$

$$\varphi = 2\phi - \sum_i \lambda_i \qquad S = (2\pi)^2 \int dt\, e^{-\varphi} \Big(\sum_i \dot{\lambda}_i^2 - \dot{\varphi}^2 \Big)$$

$$\lambda_i \to -\lambda_i,$$

i.e. R → 1/R

1.Dynamics

- Equations of Motion:

$$(2\pi)^2 e^{-\varphi}\left(\dot{\varphi}^2 - \sum_i \dot{\lambda}_i^2\right) = E$$

$$\ddot{\lambda}_i - \dot{\varphi}\dot{\lambda}_i = \frac{1}{8\pi^2} e^{\varphi} P_i$$

$$\ddot{\varphi} = \frac{1}{2}\left(\dot{\varphi}^2 + \sum_i \dot{\lambda}_i^2\right)$$

- Duality: E→E P→-P

1. Solutions

- Set pressure→0, E conserved

$$\varphi(t) = \log\left[\frac{16\pi^2/E}{t(t+C)}\right]$$

$$\lambda_i(t) = A_i + B_i \log\frac{t}{t+C}$$

$$C^2\left(1-\textstyle\sum_i B_i^2\right) = 0$$

$$e^{\varphi} \sim \frac{\text{const.}}{t^2} \qquad e^{\lambda_i} \sim \text{const}$$

1. Solutions

- Turn Pressures On: KK/Winding Modes

$$P_i \sim \begin{cases} e^{-\lambda_i} & i = 1, \ldots, m \\ 0 & i = m+1, \ldots, 9 \end{cases}$$

$$e^{\varphi} \sim \frac{1}{t^{\alpha}} \qquad e^{\lambda_i} \sim \begin{cases} t^{\beta} & i = 1, \ldots, m \\ \text{const.} & i = m+1, \ldots, 9 \end{cases}$$

$$\alpha = \frac{2m}{m+1} \qquad \beta = \frac{2}{m+1}$$

- If windings evolve to form suggested by 2+2 = 3+1, can explain why three dimensions are large.

Easther, BRG, Jackson,Kabat

We envision: Unwrapped dimensions containing radiation and hence having a usual 1/R pressure.

We also envision wrapped dimensions being near Self Dual radius where positive pressure from KK modes cancels negative pressure from windings yielding no net pressure.

Issues to Consider

- Topology:
  - Assumes spacetime is toroidal.
  - Calabi-Yau, Orbifolds, etc.
- Dynamics:
  - Realize intuitive expecations?
- M-theory:
  - Include branes. What happens?
- String Theory
  - Dilute Gas Regime

2

4 1

3

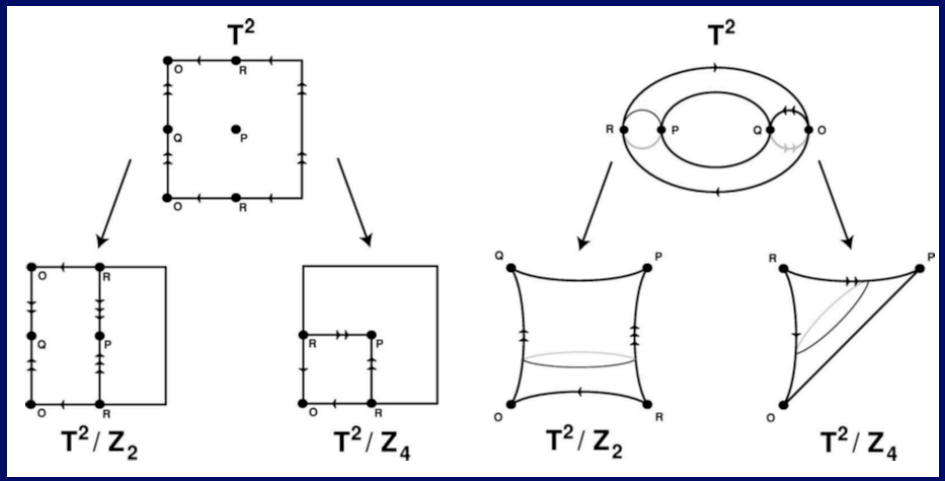
5

6

2. Orbifolds and Strings

- Generalize topology: simple orbifolds
  - No topologically protected winding modes
  - Can have long “pseudo-wound” strings
  - “Pseudo-wound” strings unwrap at orbifold fixed points.

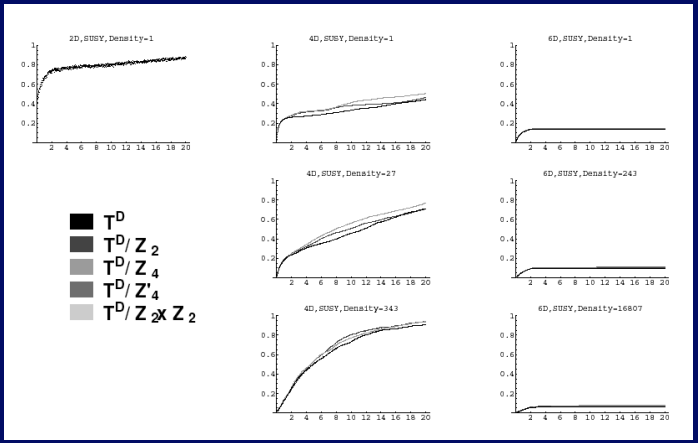
2. Pseudo-winding Modes



2. Orbifolds and Strings

- Generalize topology: simple orbifolds
  - No topologically protected winding modes
  - Can have long “pseudo-wound” strings
  - “Pseudo-wound” strings unwrap at orbifold fixed points.
- How long do these strings survive?
  - Problem now dynamical, not topological
  - Numerical simulations
  - T_{unwind} versus T_{Hubble}

2. Anisotropic Cosmological Evolution



R. Easther, BRG, M. Jackson

3. String/Brane-Gas Cosmology

- M-theory version: Wrapped Branes on T^{10}
  - Consider 5-branes, 2-branes, supergravity gas
  - Dimension counting: (5-branes irrelevant)
  - 2-branes: 3+3 = 5+1
  - 2-branes with one cycle in remaining small dimensions (effective strings): 2+2 = 3+1.
  - 3 large dimensions, 2 intermediate, 5 small.

But does it work in dynamical detail?

3. M-theory: Brane-Gas Cosmology

- Work semi-analytically on a T^{10}
  - Brane gas, not individual branes
- Low energy limit of M-theory, 11D SUGRA
  - 10 flat spatial directions
- $T_{\mu\nu}$ contains:
  - 2-branes
  - Supergravity gas
  - Brane excitations (momentum modes) modeled as thermal states on brane.

3. Background and Setup

- Metric:

$$ds^2 = -dt^2 + \sum_{i=1}^{d} (R_i(t))^2 d\theta_i^2 \qquad 0 \leq \theta_i \leq 2\pi$$

- Branes states coded by wrapping matrix.
  - N_{ij} = number of branes wrapped around (i,j) cycle
  - N_{ji} = number of (i,j) anti-branes = N_{ij}

3. Brane Action

- Nambu-Goto action - $T_2 = 1/(2\pi)^2\, l_{11}^3$

1-String Uniformly Smeared

$$T^{\mu\nu} = -T_2 \int d^3\xi \delta^{11}(X - X(\xi)) \sqrt{-\det g}\; g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu$$

$$T^\mu{}_\nu = -\frac{T_2}{2\pi R_3 \cdots 2\pi R_d} \mathrm{diag}(1,1,1,0,\ldots,0)$$

Many Strings

$$T^t{}_t = -\frac{T_2}{V}(2\pi)^2 \sum_{k \neq l} R_k R_l N_{kl}$$

$$T^i{}_i = -\frac{T_2}{V}(2\pi)^2 \sum_{k \neq i} R_k R_i (N_{ki} + N_{ik})$$

3. Equations of Motion

- Einstein equations: Wrapped branes + Supergravity gas

$$8\pi G\rho_S = \frac{1}{2}\sum_{k\neq l}\frac{\dot{R}_k\dot{R}_l}{R_kR_l} - \frac{8\pi GT_2}{V}(2\pi)^2\sum_{k\neq l}R_kR_lN_{kl}$$

$$\frac{\ddot{R}_i}{R_i} = \frac{8\pi GT_2}{V}\left[\frac{2d+1}{d(d-1)}(2\pi)^2\sum_{k\neq l}R_kR_lN_{kl} - (2\pi)^2\sum_{k\neq i}R_kR_i(N_{ki}+N_{ik})\right] + \frac{1}{2d}\sum_{k\neq l}\frac{\dot{R}_k\dot{R}_l}{R_kR_l} - \sum_{k\neq i}\frac{\dot{R}_k\dot{R}_i}{R_kR_i}$$

NOTE: FOR LATE TIME BEHAVIOUR, EXCITATIONS ON BRANES NOT RELEVANT, SO DROP.

3. Negative Pressure

- Usual isotropic case (d = 3):

$$\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{2R^2}=-\frac{8\pi GP}{2}$$

- Anisotropic case:

$$\frac{\ddot{R}_j}{R_j}+[\sum_{k\neq j}\frac{\dot{R}_k\dot{R}_j}{R_kR_j}-\frac{1}{2(d-1)}\sum_{k\neq l}\frac{\dot{R}_k\dot{R}_l}{R_kR_l}]=8\pi G[\frac{d-2}{d-1}P_j-\frac{1}{d-1}\sum_{k\neq j}P_k]$$

- Differential Growth Rate:

$$\lambda_j=\log(2\pi R_j)$$

$$\ddot{\lambda}_j-\ddot{\lambda}_m+\frac{\dot{V}}{V}(\dot{\lambda}_j-\dot{\lambda}_m)=8\pi G(P_j-P_m)$$

$$\left(\frac{\dot{V}}{V}\right)^2=\left(\sum_i\dot{\lambda}_i\right)^2=\sum_i(\dot{\lambda}_i)^2+16\pi G(\rho_S+\rho_B)$$

Volume grows monotonically

All/Most transverse dimensions wrapped implies second term in anisotropic formula kicks in substantially. For that term, negative pressure accelerates expansion so recover usual conclusion. But when **fewer transverse dimensions wrapped**, first term can dominate and this has negative pressure yielding a DECELERATION —the opposite of usual conclusion.{Eg: Consider d =10, 3 unwrapped, 7 wrapped. Consider index-J to be a wrapped dimension. Then, with all pressures the same (all 7 dimensions wrapped same way), RHS gives(8/9 – 6 (i.e.number transverse wrapped)/9)P = (2/9)P so P negative confines, does not accelerate. If all wrapped: (8/9-9/9)P=(-1/9)P implies negative pressure accelerates.

DIFFERENTIAL GROWTH RATE: P_J more negative that P_m implies RHS negative, looking like deceleration of Lambda-J vs Lambda-m.

3. Solutions

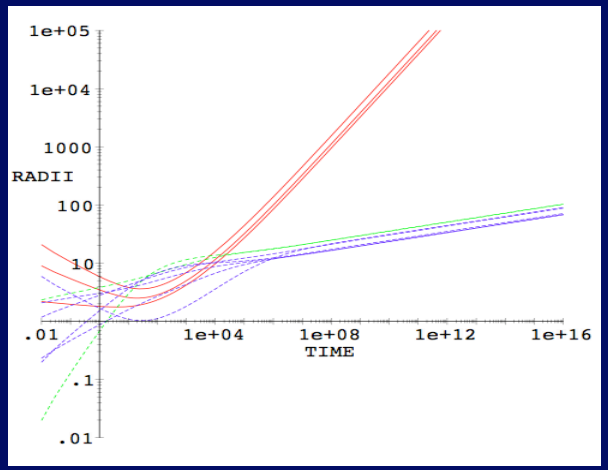
- Specify wrapping matrix
- Expectation based on **dimension counting**:
  - branes annihilate in 5D, “strings” drop out of 3 directions
- 3 unwrapped, 2 partly wrapped, 5 wrapped

$$\begin{pmatrix}
\cdot & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \cdot & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \cdot & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \cdot & 0 & N_{4,6} & N_{4,7} & N_{4,8} & N_{4,8} & N_{4,10} \\
0 & 0 & 0 & 0 & \cdot & N_{5,6} & N_{5,7} & N_{5,8} & N_{5,9} & N_{5,10} \\
0 & 0 & 0 & N_{6,4} & N_{6,5} & \cdot & N_{6,7} & N_{6,8} & N_{6,9} & N_{6,10} \\
0 & 0 & 0 & N_{7,4} & N_{7,5} & N_{7,6} & \cdot & N_{7,8} & N_{7,9} & N_{7,10} \\
0 & 0 & 0 & N_{8,4} & N_{8,5} & N_{8,6} & N_{8,7} & \cdot & N_{8,9} & N_{8,10} \\
0 & 0 & 0 & N_{9,4} & N_{9,5} & N_{9,6} & N_{9,7} & N_{9,8} & \cdot & N_{9,10} \\
0 & 0 & 0 & N_{10,4} & N_{10,5} & N_{10,6} & N_{10,7} & N_{10,8} & N_{10,9} & \cdot
\end{pmatrix}$$

We CHOOSE: radii, their velocities, and wrapping matrix. The Hamiltonian constraint then determines rho_S= energy density (and hence pressure) of supergravity gas.

About this wrapping matrix: a five dimensional subspace fluctuates large first, w/2branes having both legs wrapping 2 of those 5 dimensions, annihilating. This gives 0s in the 5 x 5 block. Note that 2 branes w/only 1 leg sitting within these 5 dimensions (and the other leg in a small dimension) do not annihilate. They look like strings in 5 d, and these generically miss. Then a 3 space in that 5 space fluctuates large. 2-branes w/one leg wrapping one of those 3 dimensions and one leg wrapping some other small dimension (effectively strings) do now annihilate, allowing these 3 dimensions to grow faster still. The annihilation of 2 branes w/one leg in 3 of the 5 dimensions, say 1,2,3 and the other leg in 4 or 5 or…10, wipes out the remaining entries in first 3 row and first 3 columns.

3. Solutions



3. General Late Time Solutions

- Assume m unwrapped directions, rest fully wrapped.
  - Sufficient: full > partial.
- At late times $R_u \sim t^{\alpha}$, $R_w \sim t^{\beta}$

m_1	0	1	2	3	4	5	6	7	8	9	10
α	–	$\frac{3}{5}$	$\frac{4}{7}$	$\frac{7}{13}$	$\frac{1}{2}$	$\frac{5}{11}$	$\frac{4}{11}$	$\frac{23}{77}$	$\frac{1}{4}$	$\frac{7}{33}$	$\frac{2}{11}$
β, γ	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{13}$	0	$-\frac{1}{11}$	$-\frac{1}{11}$	$-\frac{1}{11}$	$-\frac{1}{11}$	$-\frac{1}{11}$	–

The exponent gamma vs beta refers to partially wrapped dimensions vs fully wrapped dimensions. (A dimension is fully wrapped if the only zeroes in the wrapping matrix for that direction are for dimensions which themselves are fully Unwrapped). Notice that this table reflects our earlier observation that negative pressures are constraining—not accelerating—except when the number of wrapped dimensions is all or most, which in this table corresponds to **small** values of m For m =0,1,2,3 unwrapped dimensions, the remaining wrapped dimensions expand. For larger m, less wrapped dimensions, those who are wrapped do not expand.

3. Dynamics: Conclusion

- If achieve asymmetric wrapping matrix expected from dimension counting…
- Then dynamics leverages into hierarchy of scales.

4. Early Universe Dynamics

- Does Early Universe Dynamics Generically Yield this Favored Brane Wrapping Matrix?
  - Brane / anti-brane pairs in equilibrium with gas.
  - Find Hagedorn temperature for branes.
  - Heuristic 2-brane annihilation cross-section
  - Derive Boltzman equations for wrapping numbers
- Compute initial distributions of N_{ij} and $d\lambda_i/dt$
  - Simulate evolution numerically.
  - Number of unwound directions after freeze-out?

4. Early Universe: Details

- Evolution begins at 2-brane T_H
- Solve dynamics for multiple initial conditions.
- Require low energy limit of M-theory
  - $\lambda = \log(2\pi R) > 0$, constraint on initial volume.
  - Chosen to produce a specified initial volume.
  - $d\lambda/dt \leq 1$
  - Branes randomly distributed on cycles.
  - Total number from thermal equilibrium.

4. Early Universe: Details

- Probability distribution (d = 10)

$$S = -\frac{1}{16\pi G}\int d^{11}x\sqrt{-g}\,R = -\frac{1}{16\pi G}\int dt\,V\sum_{i\neq j}\dot{\lambda}_i\dot{\lambda}_j$$

$$H_{\text{gravity}} = \sum_i \pi_i\dot{\lambda}_i - \mathcal{L} = -\frac{V}{16\pi G}\sum_{i\neq j}\dot{\lambda}_i\dot{\lambda}_j$$

Add: Sugra Gas, Branes+Excitations

$$E_{\text{matter}} = c_S V T^{11} + \sum_{i\neq j} N_{ij}(T_2 + c_X T^3)e^{\lambda_i+\lambda_j} = \frac{V}{16\pi G}\sum_{i\neq j}\dot{\lambda}_i\dot{\lambda}_j\,.$$

$$S_{\text{matter}} = \frac{11}{10}c_S V T^{10} + \sum_{i\neq j} N_{ij}\frac{3}{2}c_X e^{\lambda_i+\lambda_j}T^2$$

$$\Gamma = \int \frac{d^d\pi\, d^d\lambda}{(2\pi)^d}\sum_{N_{ij}}\int dE_{\text{matter}}\,e^{S_{\text{matter}}}\,\delta(H_{\text{gravity}} + E_{\text{matter}})$$

- Maximize entropy: Fix V, Hamiltonian Constraint (vary T and N_{ij})

$$T = T_H \equiv \left(\frac{2T_2}{c_X}\right)^{1/3}$$

$$\mathcal{N}_{\text{eq}} \equiv \sum_{i\neq j} N_{ij}e^{\lambda_i+\lambda_j} = \frac{V}{3T_2}\left(\frac{1}{16\pi G}\sum_{i\neq j}\dot{\lambda}_i\dot{\lambda}_j - c_S T_H^{11}\right)$$

- Hagedorn Phase: Entropically favored ($s = S_{\text{matter}}/V$)

$$s = \frac{1}{T_H}\left(\frac{1}{d}c_S T_H^{11} + \frac{d(d-1)}{16\pi G}\dot{\lambda}^2\right)$$

$$\dot{\lambda} > \left(\frac{16\pi G c_S T_H^{11}}{d(d-1)}\right)^{1/2} \approx 0.502$$

For Hagedorn phase, this assumes all lambda equal. **Point is: Entropy favors larger velocity for radii; if velocity is larger than .502,then get nonzero equilibrium value for surface area in membranes, N_eq. When N_eq drops below zero, only the supergravity gas remain in thermodynamic equilibrium, so we have left Hagedorn and entered radiation phase.**

Note: When add energy to Hagedorn phase, temperature does not rise. Instead, produce more 2-branes. In reverse, as energy drops in Hagedorn, you lose 2-branes. When all 2-branes gone, further decrease in energy is accompanied by decrease in temperature, causing temperature to drop below T_H, exiting Hagedorn phase.

4. Early Universe: Details

- Evolution via EOM-Boltzman Coupled System

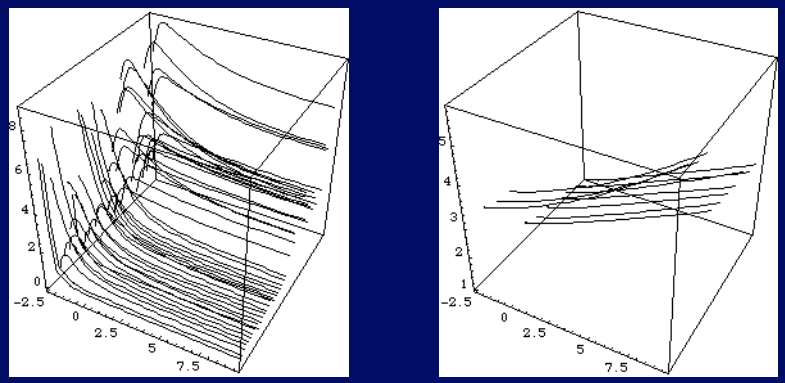
$$\frac{d}{dt}N_{ij} = -\frac{16\pi G\, T_2^{4/3}\, f\, e^{2(\lambda_i+\lambda_j)}}{V}\left((N_{ij})^2 - (N_{ij}^{\rm eq})^2\right)$$

$$f(v) = \frac{2}{1-v^2}$$

- Algorithm: Evolve λ's and N's for many sets of initial conditions.
  - Fix Volume
  - Randomly set λ's; apply overall scale to match V
  - Randomly set dλ/dt's according to entropy measure
  - Calculate total equilibrium area $\mathbf{A_{eq}}$ of wrapped branes
  - Spread equally among cycles $N_{ij}^{eq} = (1/45)[\mathbf{A}_{eq}]\exp[-\lambda_i-\lambda_j]$
  - Randomly choose initial N_{ij}'s to carry this area
  - If any N_{ij} drops below .5, it is unwound.

The variable v is the transverse velocity of the wrapped membrane.

4. Sample Dynamics

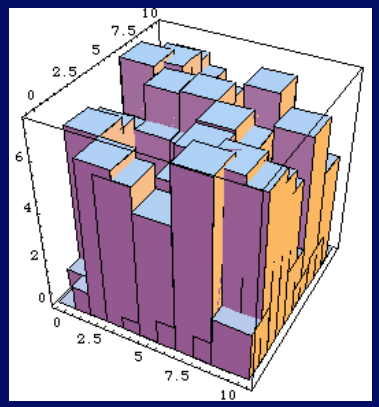


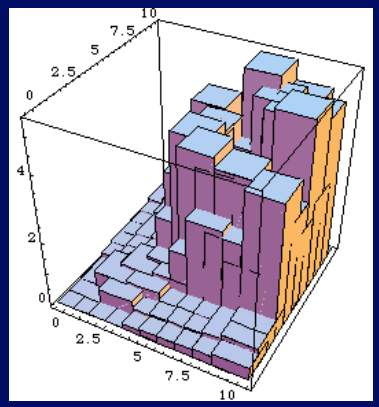
Initial log(V) = 18

Left plot shows evolution of 45 wrapping numbers.

Right plot shows evolution of 10 toroidal dimensions

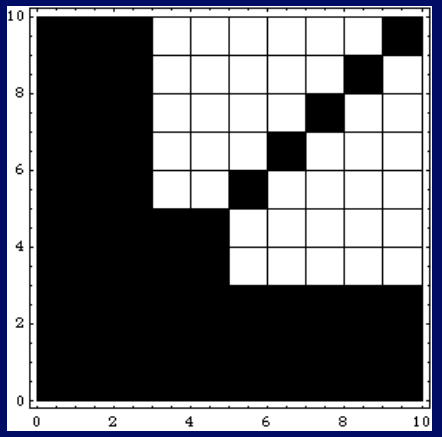
4. Winding Evolution





Initial log(V) = 18

4. Unwound Directions

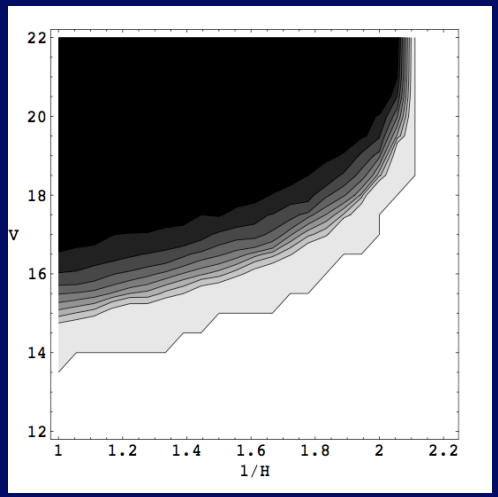


Black=N_{ij}<1

4. Distribution of Dimensionality

- Some runs Realize expectation, but most don't.

(Black = No decompactification; White = All decompactify; all



In this plot, 1/H is lambda dot—all lambda dots set equal for this visualization---only weak dependence on their individual values in other runs. The horizontal axis is really 1/lambda-dot (since lambda-dot = R-dot/R = H). On the far right, lambda-dot is too small to begin in Hagedorn, i.e. the equilibrium number of wrapped branes has dropped to zero, so everybody decompactifies.

Larger iniital volume have a harder time shedding wrapped membranes: Competing effects are at work: Large volume implies more winding. But also, at first means more efficient annihilation (due to area-squared factor in cross section). As numbers of winding begins to drop, efficiency drops, branes freeze out.

4. Lesson/Question

- Detailed dynamics need not confirm naïve dimension counting arguments.
- **Issue**: Due to spatial expansion, branes freeze out.
- Suggestion: String theory corner of moduli space.

$$N_{i,10} = N_{10,i} \neq 0; N_{i,j} = 0$$

$$R_i \approx t^{1/4}, R_{10} \approx t^{-1/2}$$

$$ds_M^2 = e^{-2\phi/3} ds_{ST}^2 + e^{4\phi/3} (dx^{10})^2, e^{\phi} \approx t^{-3/4}$$

$$\Rightarrow ds_{ST}^2 \approx \text{constant}$$

Important note: This slide emphasizes the case where no dimensions decompactify (the balck regions on last page)

Issues to Consider

- Topology: 2
  - Assumes spacetime is toroidal.
  - Calabi-Yau, Orbifolds, etc.
- Dynamics: 4 1
  - Realize intuitive expecations?
- M-theory: 3
  - Include branes. What happens?
- String Theory 5 6
  - Dilute Gas Regime

5. String Dynamics

- Recall from section 1:

$$P_i \sim \begin{cases} e^{-\lambda_i} & i = 1, \ldots, m \\ 0 & i = m+1, \ldots, 9 \end{cases}$$

$$e^{\varphi} \sim \frac{1}{t^{\alpha}} \qquad e^{\lambda_i} \sim \begin{cases} t^{\beta} & i = 1, \ldots, m \\ \text{const.} & i = m+1, \ldots, 9 \end{cases}$$

$$\alpha = \frac{2m}{m+1} \qquad \beta = \frac{2}{m+1}$$

- If windings evolve to form suggested by 2+2 = 3+1, can explain why three dimensions are large.

Easter, BRG, Jackson,Kabat

5. String Dynamics

- Thermodynamics: Two Phases
  - Hagedorn Phase: $T_H = \frac{1}{\pi\sqrt{8}}$ $S(E) = E/T_H$ $P_i = -\frac{\partial F}{\partial \lambda_i} = 0$
    - Number of strings with energy ε, winding **w:** Deo, Jain, Tan

$$D(\epsilon, \mathbf{w}, E) = \frac{N}{\epsilon} u(\epsilon, E)^{d/2} e^{-u(\epsilon,E)\mathbf{w}^T A^{-1}\mathbf{w}/4}$$

$$N = \frac{(2\sqrt{\pi})^{-d}}{\sqrt{\det A}} \qquad u(\epsilon, E) = \frac{E}{\epsilon(E-\epsilon)} \qquad A_{ij} = \frac{1}{4\pi^2 R_i^2}\delta_{ij}$$

$$\langle W_i \rangle = \int_0^{E/9} d\epsilon \int_0^{\infty} dw_i \; w_i D(\epsilon, w_i, E) = \frac{\sqrt{E}}{12\sqrt{\pi} R_i}$$

$$\langle N_i \rangle = \frac{\sqrt{E} R_i}{12\sqrt{\pi}}$$

5. String Dynamics

- Thermodynamics: Two Phases
  - Radiation Phase: d-dimensions 'unfrozen' (smaller/larger than self-dual radius)

$$V_d = \prod_{i=1}^{d} 2\pi e^{|\lambda_i|}$$

$$E = c_d V_d T^{d+1}$$

$$S = \frac{d+1}{d} c_d V_d T^d$$

$$P_i = -\frac{\partial F}{\partial \lambda_i} = \begin{cases} \text{sign}(\lambda_i) E/d & i = 1, \ldots, d \\ 0 & i = d+1, \ldots, 9 \end{cases}$$

$$\langle N_i \rangle = \frac{1}{2} P_i R_i$$

$$\langle W_i \rangle = 0$$

This is a note

5. Boltzman Equations

$$\frac{dN_i}{dt} = -\frac{1}{\pi} e^{\varphi} \langle \epsilon_i \rangle^2 (N_i^2 - \langle N_i \rangle^2)$$
$$\frac{dW_i}{dt} = -\frac{1}{\pi} e^{\varphi} \langle \delta_i \rangle^2 (W_i^2 - \langle W_i \rangle^2)$$

Hagedorn phase : $\langle \epsilon_i \rangle = \dfrac{E}{9\langle N_i \rangle}$ (momentum modes)

$\langle \delta_i \rangle = \dfrac{E}{9\langle W_i \rangle}$ (winding modes)

Radiation phase : $\langle \epsilon_i \rangle = 1/R_i$ (momentum modes)

$\langle \delta_i \rangle = R_i$ (winding modes) .

5. Boltzman-Einstein Evolution

- Algorithm: Evolve N's, W's, λ's for many sets of initial conditions.
  - Fix Volume V; choose random λs compatible with V.
  - Fix φ
  - Fix dφ/dt = -1
  - Choose dλ/dt's from Gaussian distribution:

$$S = E/T_H = (2\pi)^2 e^{-\varphi}(\dot{\varphi}^2 - \sum_i \dot{\lambda}_i^2)/T_H$$

  - Determine E; determine Hagedorn vs Radiation phase.
  - Choose momenta/winding-equilibrium (Hag. or rad.) plus fluctuations:

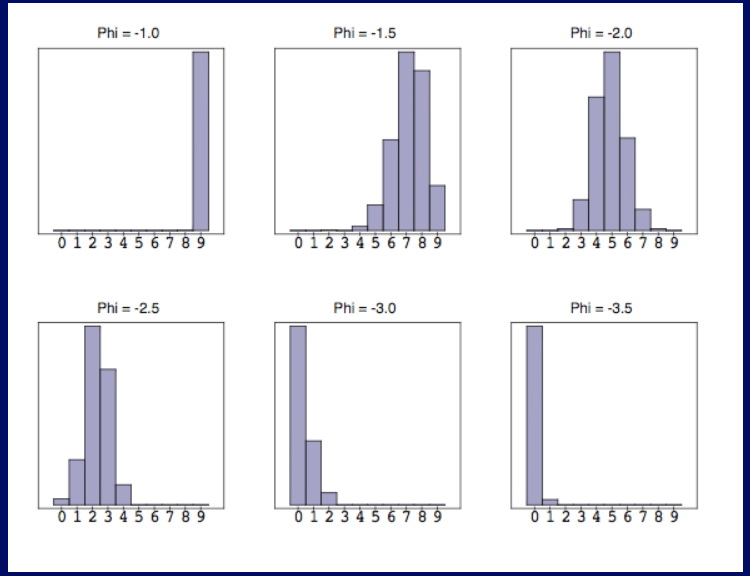
$$\Delta N_i \approx \sqrt{\langle N_i \rangle} \qquad \Delta W_i \approx \sqrt{\langle W_i \rangle}$$

  - Set pressures:

$$P_i = 2\left(N_i e^{-\lambda_i} - W_i e^{\lambda_i}\right)$$

Note: One odd thing to bear in mind: The Gaussian distribution comes from Hagedorn relation between entropy and energy. However, the initial data may turn out to be such that we begin in radiation phase. If so the velocity distribution is not quite relevant…

5. Results



Horizontal Axis: Number of UNWRAPPED DIMENSIONS at late times

Vertical Axis: Number of runs that have the corresponding number of unwrapped dimensions at late times.

EVER MORE NEGATIVE DILATON MEANS EVER MORE ENERGY IN MATTER—IE. MORE WINDING MODES. HARDER TO GET RID OF.

Nothing Special about 3 unwrapped dimensions. Still has an ALL or NOTHING Character

6. Dilute Gas/Initial Conditions

- Heart of problem:
  - Freeze Out (Leaves many wrapped dimensions)
  - Increase φ (to help interactions), less winding implies all decompactify.
  - Insufficient distinction in cross section for 3 vs higher spatial dimensions.
- Two other issues:
  - Have assumed uniform distribution of winding in transverse dimensions. Often too dilute to be true.
  - Have chosen initial conditions from equilibrated values.
- Proposal:
  - Incorporate dilute Gas Effects
  - Set IC in Hagedorn phase—then fluctuate.

6. Dilute Gas

- Use impact parameter formalism for cross section: (Amati, Ciafoloni,Veneziano)
  - D>3 Cross section: b = impact, Δx = string “width”

$$\Gamma_W \sim \left(\frac{e^{\phi+2\lambda}}{V}\frac{2\pi e^{\lambda}}{\Delta x}\right)e^{-b^2/\Delta x^2}$$

  - D<3 No impact parameter; use previous formula

6. Algorithm: Initial Data

Need 6 initial conditions: $\varphi_0, \dot{\varphi}_0, \lambda_0, \dot{\lambda}_0, W_0, K_0$

$\dot{\varphi}_0 = -1$

At self-dual radius:

Choose $\varphi_0 \longrightarrow E_{sd} = (2\pi)^2 e^{-\varphi_0}\dot{\varphi}_0^2 \;\; (\dot{\lambda} = 0) \;\; \longrightarrow W_{sd} = K_{sd} = \frac{1}{12}\sqrt{\frac{E_{sd}}{\pi}}$

Fluctuation:

$\rightarrow$ Get $\dot{\lambda}_0$ from $e^S \sim e^{-\dot{\lambda}_0^2/\sigma^2} \longrightarrow E = (2\pi)^2 e^{-\varphi_0}(\dot{\varphi}_0^2 - d\dot{\lambda}_0^2)$

$\rightarrow$ Choose $\lambda_0 \longrightarrow V_d = (2\pi)^d e^{d\lambda_0}$

$E, V_d \longrightarrow$ Determine Equilibrium Thermodynamics:

$$\rho = \frac{E}{V_d} \begin{cases} \geq c_d T_{\text{Hag}}^{d+1} \longrightarrow \text{Hagedorn phase} \begin{cases} \langle W \rangle = \frac{1}{12}\sqrt{\frac{E}{\pi}} e^{-\lambda} & W_0 = \text{Random}\big(\text{Max}\{0.5, \langle W \rangle\}, W_{sd}\big) \\ \langle K \rangle = \frac{1}{12}\sqrt{\frac{E}{\pi}} e^{\lambda} & K_0 = \text{Random}\big(K_{sd}, \langle K \rangle\big) \end{cases} \\ < c_d T_{\text{Hag}}^{d+1} \longrightarrow \text{Radiation phase} \begin{cases} \langle W \rangle = 0 & W_0 = \text{Random}\big(0.5, \text{Min}\{W_{sd}, \frac{E}{2d} e^{-\lambda}\}\big) \\ \langle K \rangle = \frac{E}{2d} e^{\lambda} & K_0 = \text{Random}(0,1) \times \left(\frac{E - E_W}{2d}\right) e^{\lambda} \end{cases} \end{cases}$$

6. Algorithm: Einstein-Boltzman

Start E.O.M - Dilaton gravity with $\dot{W} = -\Gamma_W \left(W^2 - \langle W\rangle^2\right), \quad \dot{K} = -\Gamma_K \left(K^2 - \langle K\rangle^2\right)$

$\Gamma_0 \equiv e^{\varphi+2\lambda} \sim g_s^2 s/V, \quad \Delta x^2 \equiv \mathrm{Log}(s) = 2\lambda, \quad \Delta t_{\rm cross} \equiv \frac{2\pi e^\lambda}{v_W}$

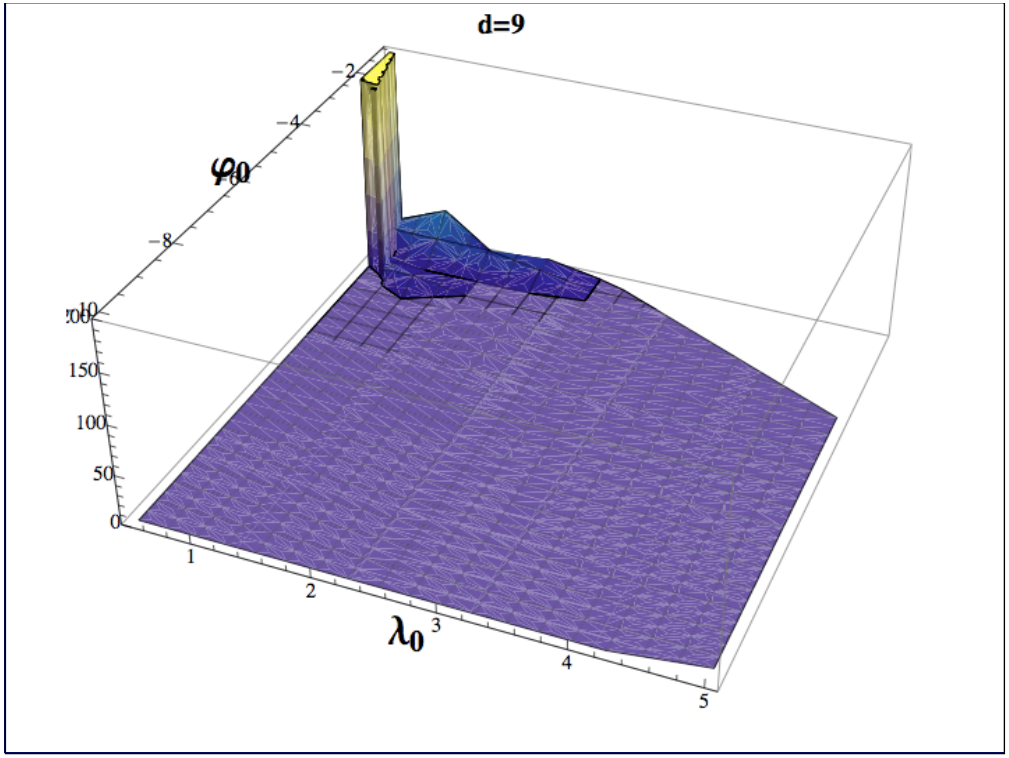
- Hagedorn phase $\rightarrow \Gamma_W = \mathrm{Energy} \times \Gamma_0$, $\Gamma_K = \mathrm{Energy} \times \mathrm{T\text{-}dual}(\Gamma_0)$
- Radiation phase
  - $\frac{2\pi e^\lambda}{\Delta x} < 1 \rightarrow \Gamma_W = \Gamma_0, \ \Gamma_K = \mathrm{T\text{-}dual}(\Gamma_0)$
  - $\frac{2\pi e^\lambda}{\Delta x} > 1 \rightarrow \Gamma_K = \mathrm{T\text{-}dual}(\Gamma_0)$
    - $d = 3:$ $\begin{cases} (\text{Time to restore } \frac{2\pi e^\lambda}{\Delta x} < 1)|_{\Gamma_W=0} < \Delta t_{\rm cross} \rightarrow \text{ consistent } \rightarrow \text{ continue} \\ (\text{Time to restore } \frac{2\pi e^\lambda}{\Delta x} < 1)|_{\Gamma_W=0} > \Delta t_{\rm cross} \rightarrow \Gamma_W = \Gamma_0 \end{cases}$
    - $d > 3:$ Every $\Delta t_{\rm cross}$, set $b^2 = \mathrm{Random}(0,1) \times \left(\frac{2\pi e^\lambda}{W}\right)^2 \rightarrow \Gamma_W = \Gamma_0 \times \mathrm{Min}\left[1, \left(\frac{2\pi e^\lambda}{\Delta x} e^{-b^2/\Delta x^2}\right)^{d-3}\right]$

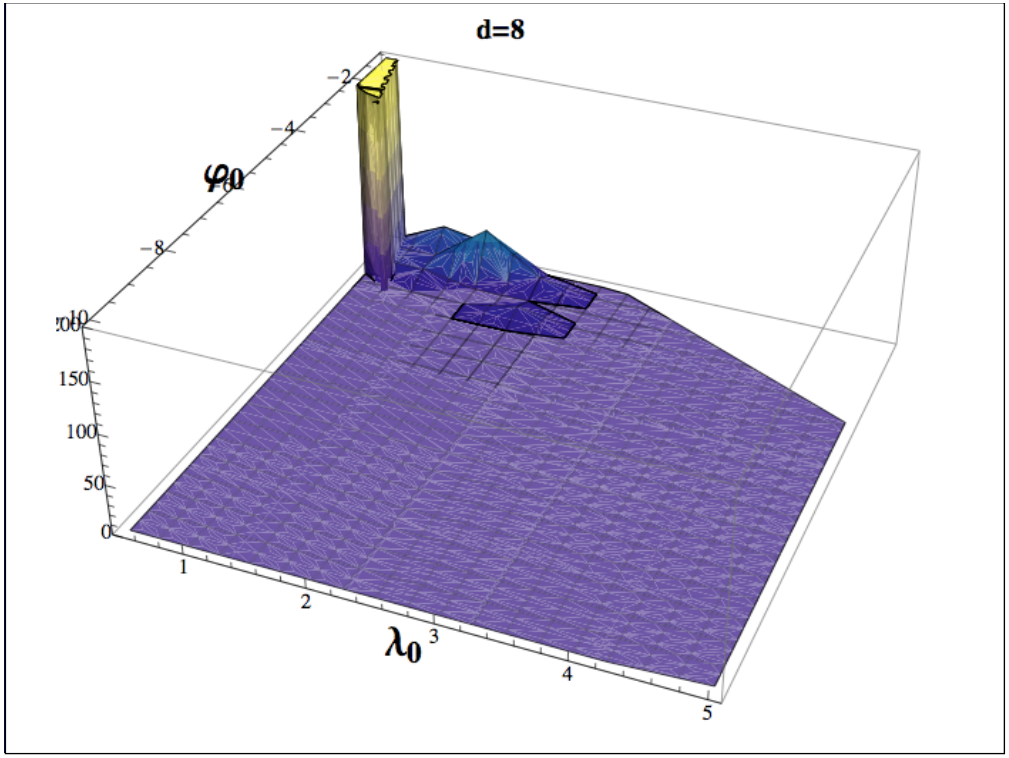
Finish when either $W = 0 (< 0.5)$ or $\Gamma_W W < \dot{\lambda}$ (Freeze-out)

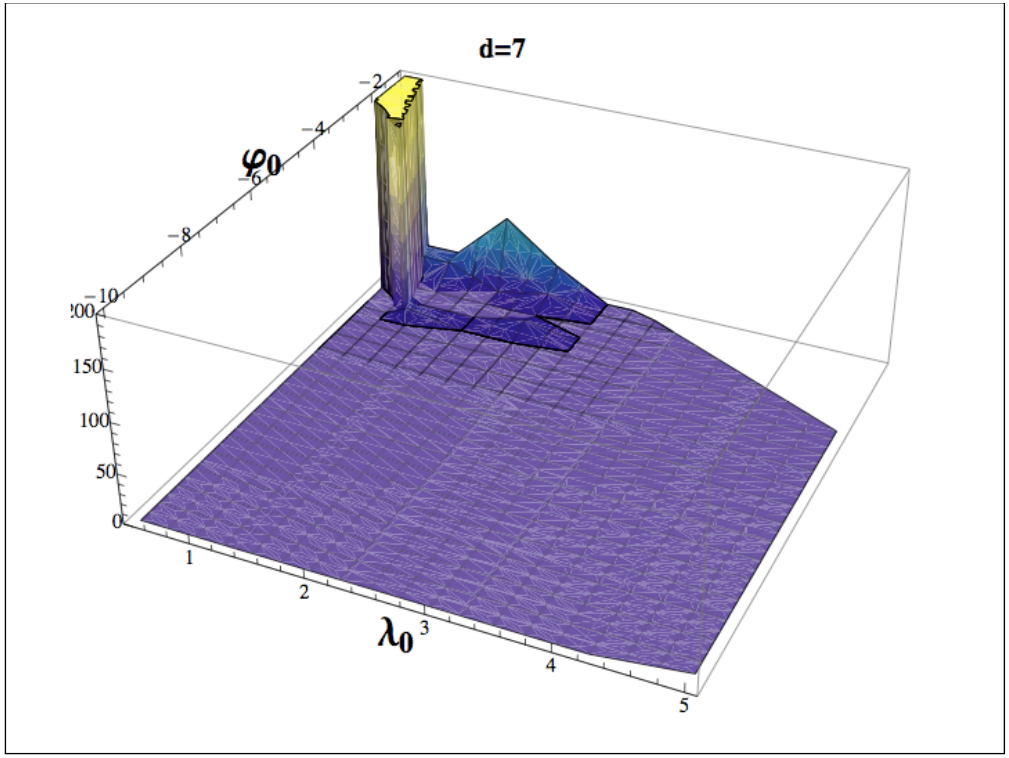
Einstein here is really Einstein-Dilaton

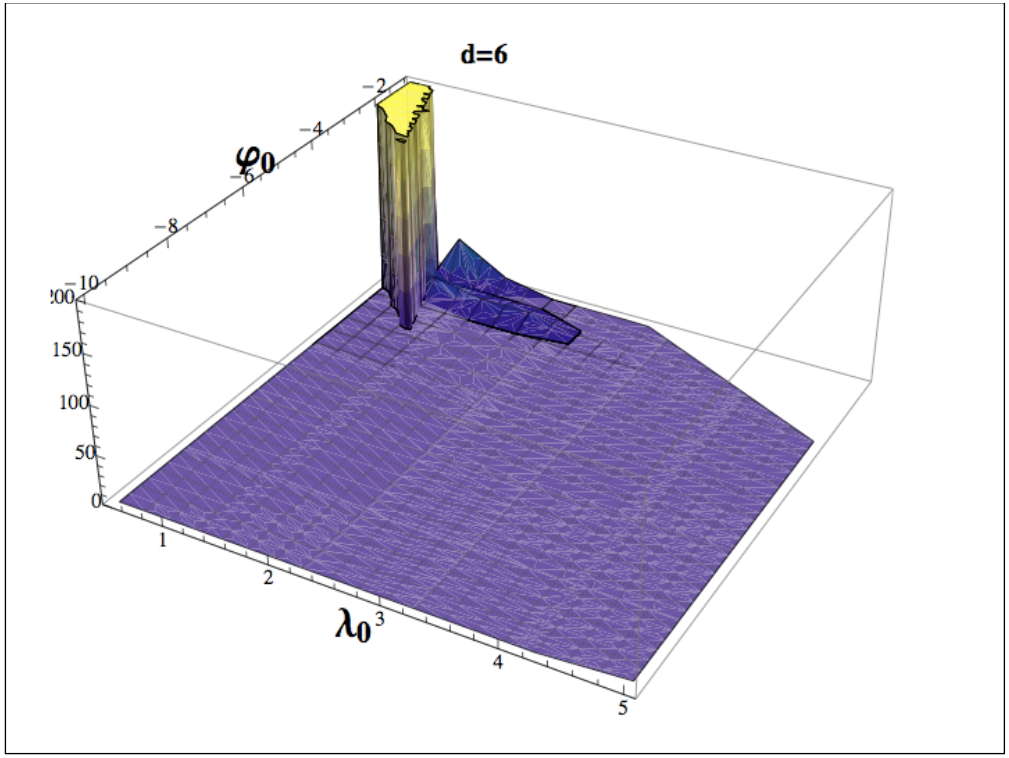
6. Results

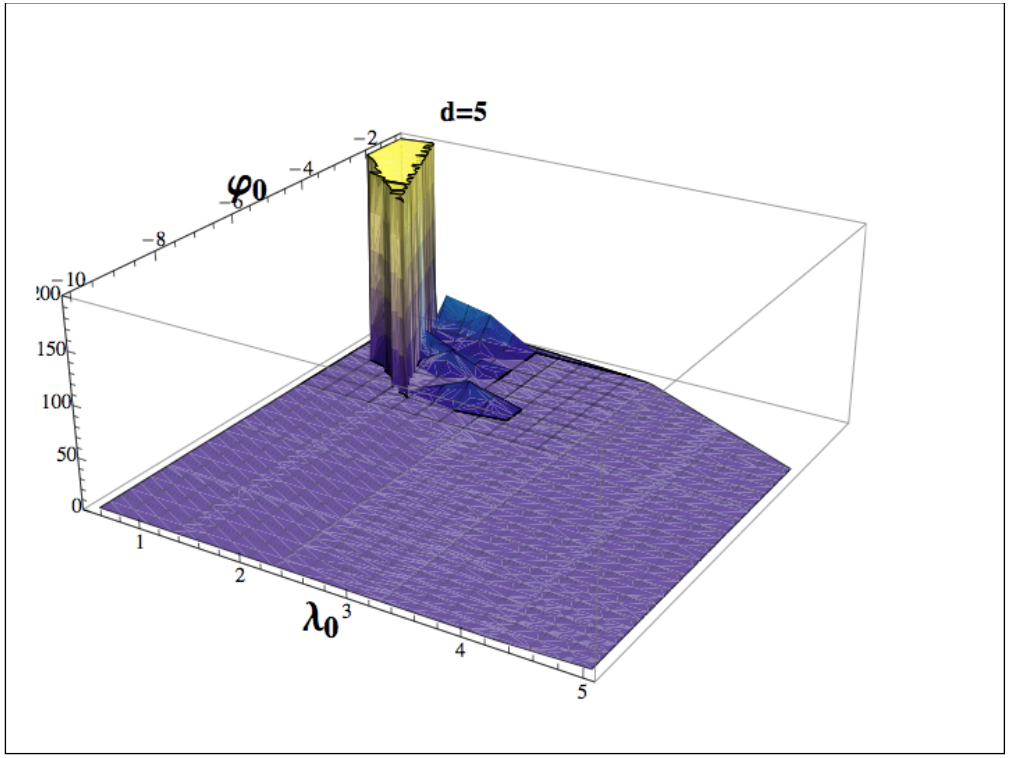
- For various numbers of dimensions that experience a thermal fluctuation in size, compare number of runs, for each choice of initial data, that decompactify.

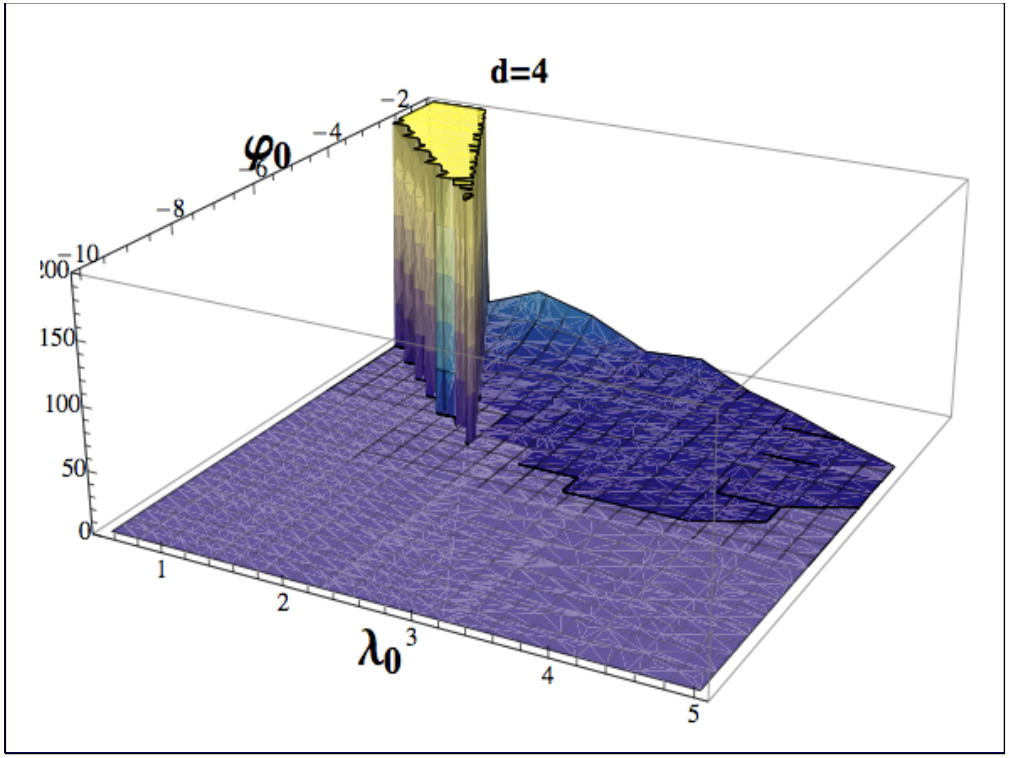

d=9
−2
−4
φ0
−8
200
150
100
50
0
1
2
λ0
3
4
5

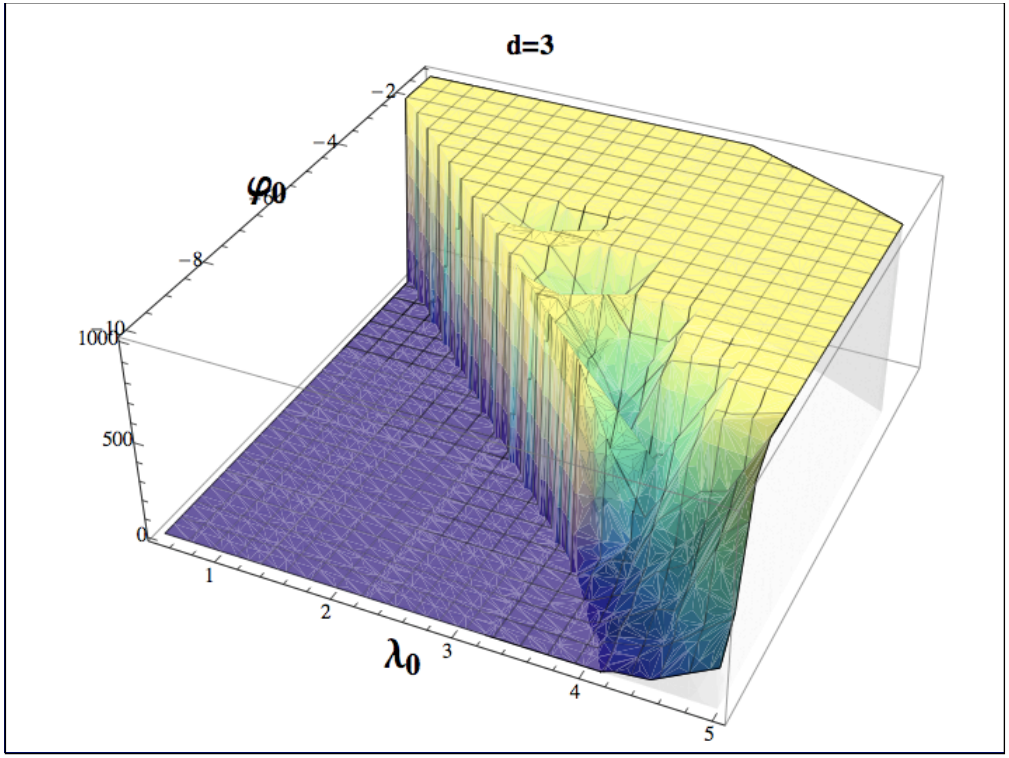

d=8
φ_0
−2
−4
−8
150
100
50
0
1
2
3
4
5
λ_0

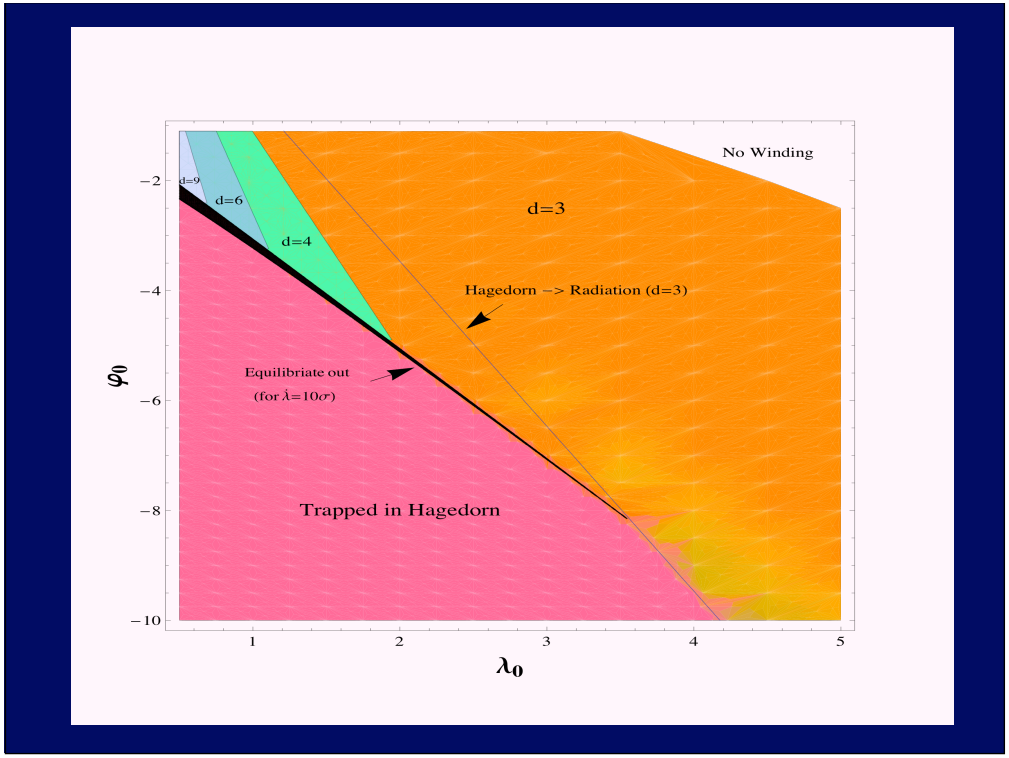

d=7
φ_0
−2
−4
−6
−8
−10
200
150
100
50
0
1
2
3
4
5
λ_0


d=6
φ_0
−2
−4
−6
−8
−10
200
150
100
50
0
1
2
$\lambda_0{}^3$
4
5


d=5
φ_0
−2
−4
−6
−8
−10
200
150
100
50
0
1
2
$\lambda_0{}^3$
4
5


d=4
φ_0
−2
−4
−6
−8
−10
200
150
100
50
0
1
2
3
4
5
λ_0


d=3
φ_0
−2
−4
−6
−8
10
1000
500
0
1
2
3
4
5
λ_0


No Winding
d=9
d=6
d=4
d=3
Hagedorn −> Radiation (d=3)
Equilibriate out
(for λ=10σ)
Trapped in Hagedorn
−2
−4
−6
−8
−10
1
2
3
4
5
φ_0
λ_0

Conclusions

Positive	Negative
Trivial Fundamental Group	M-theory Freeze Out from Volume growth. M-theory decompactify from too few brane wrapping. ALL or NOTHING
Brane/String Dynamics leads to wrapping/winding numbers suggested by classical geometrical reasoning.	String Theory Freeze out from Dilaton drop String-theory decompactify from too few winding modes ALL or NOTHING
Modify Cross Section from consideration of dilute gas regime; Set IC at Hagedorn, fluctuate from there. Clear distinction between d=3 and d>3.	