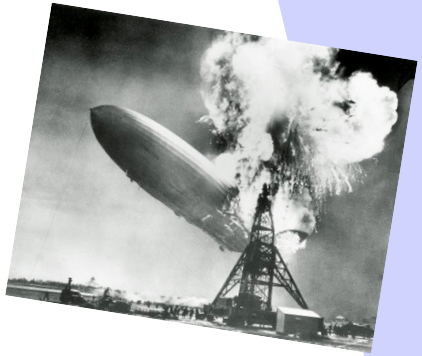
Cosmology and Inductive Inference

A Bayesian Failure



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This Talk

Bayesian probabilistic analysis is the wrong formal tool for investigating some evidential relations in cosmology.

Neutral support conflated with disfavoring evidence.

Fragments of inductive logics that tolerate neutral support displayed.

Non-probabilistic state of *completely* neutral support.

Artifacts are introduced by the use of the wrong inductive logic.

Doomsday argument.

Probabilities are recoverable from multiverses if there is a randomizer.

Self-sampling Assumption.

The Surprising Analysis

!!! The Surprising Analysis *(informal version)*

1. **Establishment** that prior theory is neutral with regard to a particular cosmic feature.

Parameter uniquely favorable to our particular form of life.
Spatial geometry so close to flat.

2. **The specific** value observed for the feature is surprising and in need of explanation.

3. **The provision** of the explanation to which we should infer.

Inflationary cosmology.
Anthropic reasoning over multiverses.

!!! Surprising Analysis *(Bayesian Version)*

i A physical parameter k is left indeterminate by our background knowledge "B".

Probability distribution $p(k|B)$ widely spread over the admissible values of k. Observed value k_{obs} has low probability:

$p(k_{obs}|B)$ is small.

ii Were some theory T to be the case, then the probability of k_{obs} would be much higher.

The likelihood

$p(k_{obs}|T\&B)$ is large.

iii Bayes' theorem: evidence k_{obs} lends strong support to the theory T.

$$\frac{p(T\,|k_{obs}\&B)}{p(T\,|B)} = \frac{p(k_{obs}|T\&B)}{p(k_{obs}|B)}$$

Does this Explication by Probabilities Succeed?

Can we really supply credible values for: $p(k_{obs}|B)$ $p(k_{obs}|\text{not-}T\&B)$

Underlying motivation

Logic of physical chances → Logic of all evidence

Neutral Evidential Support

Neutral Evidential Support

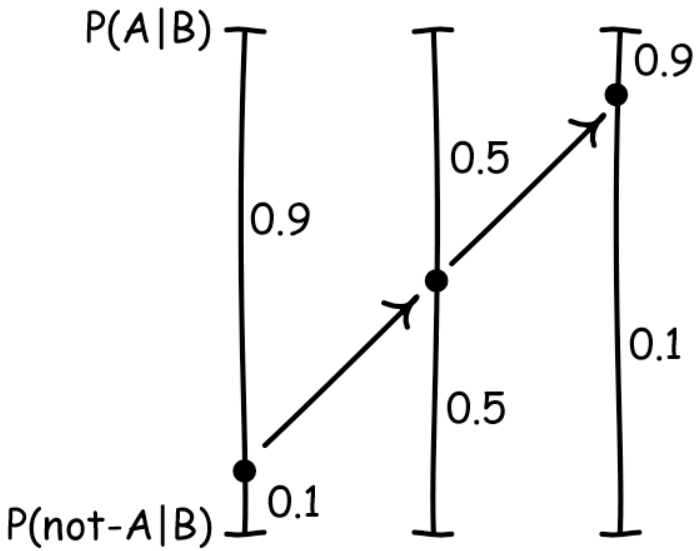
Evidence has no bearing either way. Indifference. Hypothesis left indeterminate.
Case of Surprize Analysis.

versus

Disfavoring Evidence

Evidence speaks against hypothesis.
Case of low probability.

Probabilities from 1 to 0 span support to disfavor



$$P(A|B) + P(\text{not-}A|B) = 1$$

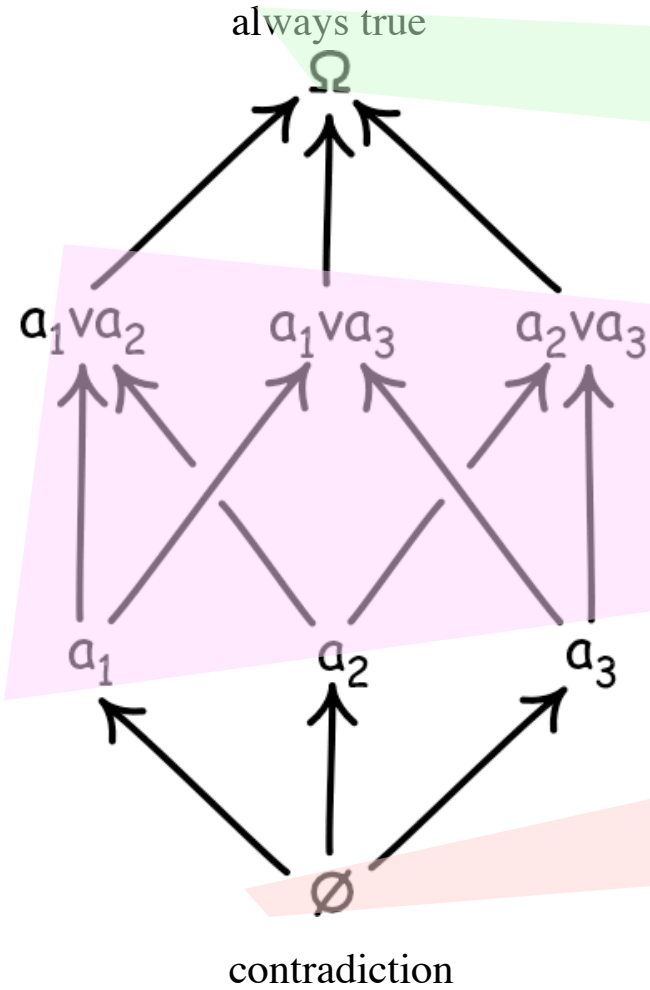
this is small → this must be large

A dis-favored ← not-A favored

No neutral probability value

Representing *Completely* Neutral Support

[A|B] = support
A accrues from B



$[\Omega|B] = 1$ true

[any contin -gent A |B] = I indifference ignorance

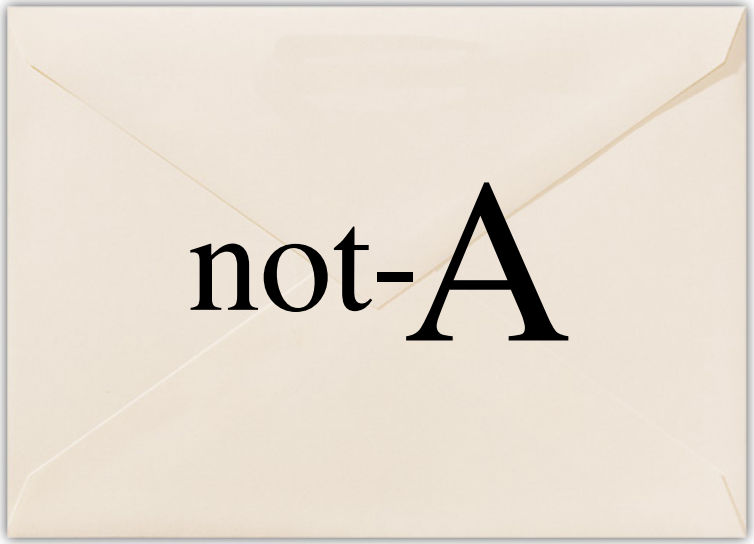
$[\varnothing|B] = 0$ false

John D. Norton, "Ignorance and Indifference." *Philosophy of Science*, **75** (2008), pp. 45-68.
"Disbelief as the Dual of Belief." *International Studies in the Philosophy of Science*, **21**(2007), pp. 231-252.

Justification (briefly)

I. Invariance under Negation

Background evidence is completely neutral with respect to proposition A.



II. Invariance under disjunctive refinement

= the real import of the "paradoxes of the principle of indifference"

Representation of completely neutral support is unchanged if propositions are replaced by their negations.

+

monotonicity

=

all contingent propositions accrue support I

mad dog

Neutrality and Disfavor or Ignorance and Disbelief

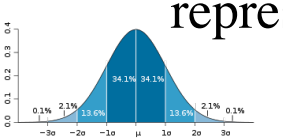
Bruno de Finetti

Objective Bayesianism
degrees of *support*

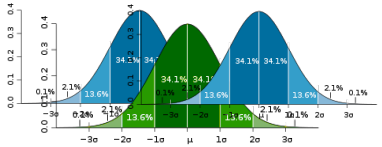
Subjective Bayesianism
degrees of *belief*

In each evidential situation,

Only one conditional probability correctly represents the import of evidence.



Many conditional probability represents opinion + the import of evidence.



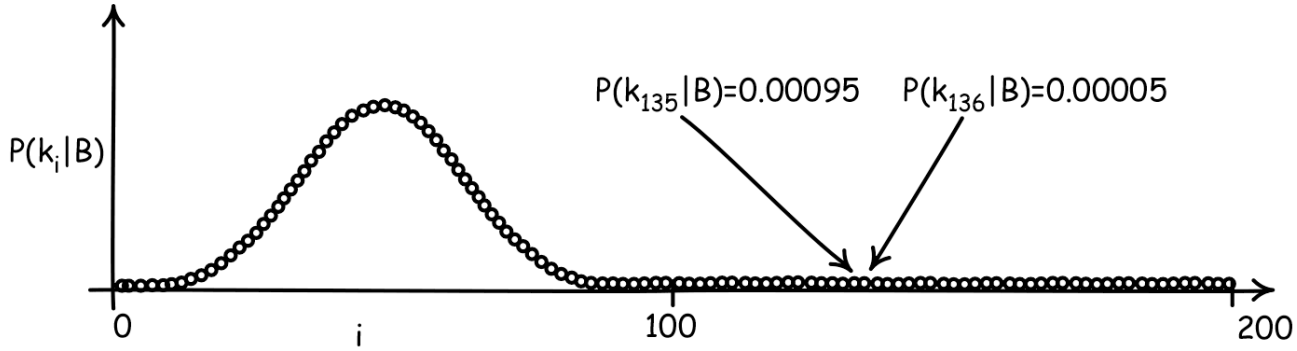
Initial "informationless" priors?

Impossible.
No probability measure captures complete neutrality.

Pick any.
They merely encode arbitrary opinion that will be wash out by evidence.

Pure Opinion Masquerading as Knowledge

1. Subjective Bayesian sets arbitrary priors on $k_1, k_2, k_3, \ldots$ Pure opinion.



2. Learn evidence

$E = k_{135}$ or k_{136}

3. Apply Bayes' theorem

$$\frac{P(k_{135}|E\&B)}{P(k_{136}|E\&B)} = \frac{P(k_{135}|B)}{P(k_{136}|B)} = \frac{0.00095}{0.00005}$$

$P(k_{135}|E\&B) = 0.95$

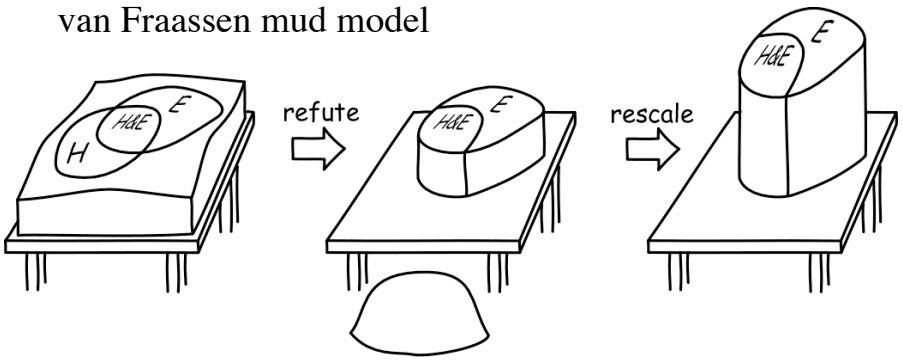
$P(k_{136}|E\&B) = 0.05$

Inductive Logics that Tolerate

Neutrality of Support

Refute and Rescale Dynamics

van Fraassen mud model



Bayesian conditionalization.

If

T_1 entails E. T_2 entails E.

$P(T_1|B) = P(T_2|B)$

then

$P(T_1|E\&B) = P(T_2|E\&B)$

Postulate same rule in new inductive logic

Conditionalizing from Complete Neutrality of Support

If

T_1 entails E. T_2 entails E.

$[T_1|B] = [T_2|B] = I$

then

$[T_1|E\&B] = [T_2|E\&B]$

Pure Opinion Masquerading as Knowledge *Solved*

"Priors" are completely neutral support over all values of k_i.

$$[k_1|B] = [k_2|B] = [k_3|B] = \ldots = [k_{135}|B] = [k_{136}|B] = \ldots = I$$

No normalization imposed.

$$[k_1|B] = [k_1 \text{ or } k_2|B] = [k_1 \text{ or } k_2 \text{ or } k_3|B] = \ldots = I$$

Apply rule of conditionalization on completely neutral support.

$$E = k_{135} \text{ or } k_{136}$$

$$[k_{135}|B] = [k_{136}|B] = I \quad \Rightarrow \quad [k_{135}|E\&B] = [k_{136}|E\&B]$$

Nothing in evidence discriminates between k_{135} or k_{136}.

Bayesian result of support for k_{135} over k_{136} is an **artifact** of the inability of a probability measure to represent neutrality of support.

The Doomsday Argument

Doomsday Argument *(Bayesian analysis)*

time = 0

we learn time t has passed

time of doom T

What support does t give to different times of doom T?

Bayes' theorem

$$p(T|t\&B) \sim p(t|T\&B) \,.\, p(T|B)$$

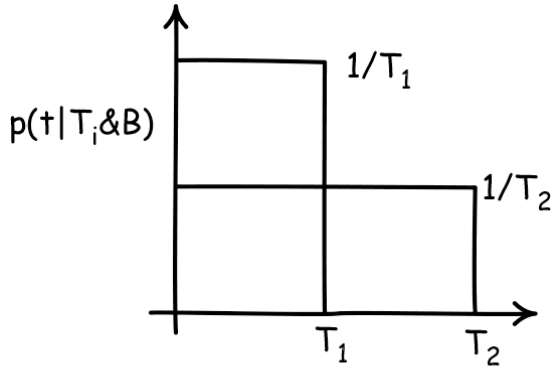
Compute likelihood by assuming t is *sampled uniformly* from available times 0 to T.

$$p(t|T\&B) = 1/T$$

For later: which is the right "clock" in which to sample uniformly? Physical time T? Number of people alive T'?…

$$p(T|t\&B) \sim 1/T$$

Support for early doom



Variation in likelihoods arise entirely from normalization.

Entire result depends on this normalization.

Entire result is an artifact of the use of the wrong inductive logic.

Doomsday Argument *(Barest non-probabilistic reanalysis.)*

time = 0

we learn time t has passed

time of doom T

What support does t give to different times of doom T?

Take evidence E is just that T>t.

T_1>t entails E. T_2>t entails E.

$E = T>t$

$[T_1|B] = [T_2|B] = I$

Apply rule of conditionalization on completely neutral support.

$[T_1|E\&B] = [T_2|E\&B]$

The evidence fails to discriminate between T_1 and T_2.

Doomsday Argument *(Bayesian analysis again)*

time = 0

we learn time t has passed

time of doom T

Consider only the posterior

$$p(T|t\&B)$$

Require invariance of posterior under changes of units used to measure times T, t.

Invariance under T'=AT, t'=At

Days, weeks, years? Problem as posed presumes no time scale, no preferred unit of time.

Unique solution is the "Jeffreys' prior."

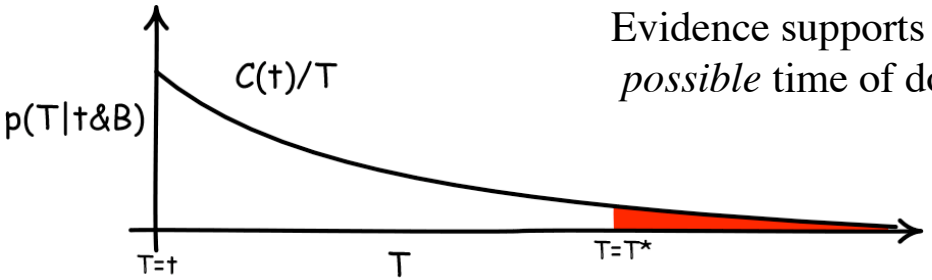
$$p(T|t\&B) = C(t)/T$$

Disaster! This density cannot be normalized.

Infinite probability mass assigned to $T>T^*$, no matter how large.

Evidence supports *latest possible* time of doom.

What support does t give to different times of doom T?



A Richer Non-Probabilistic Analysis

time = 0

we learn time t has passed

time of doom T

What support does t give to different times of doom T?

Consider the non-probabilistic degree of support for T in the interval

$$[T_1,T_2|t\&B]$$

Presume that there is a "right" clock-time in which to do the analysis, but we don't know which it is. So we may privilege no clock, which means we require invariance under change of clock:

$$T' = f(T),\ t' = f(t),$$

for strictly monotonic f.

$$[T_1,T_2|t\&B] = [T_3,T_4|t\&B] = I$$

for all T_1, T_2, T_3, T_4

Bringing Back Probabilities?

Which is the Right Inductive Logic?

Material theory of induction:

Inductive inferences are not warranted by universal schema, but by locally prevailing facts.

The contingent facts prevailing in a domain dictate which inductive logic is applicable.

A Warrant for a Probabilistic Logic

Ensemble

+

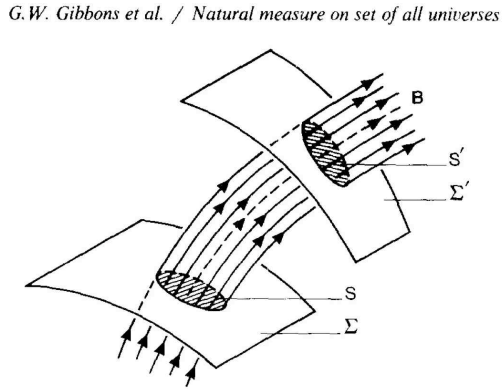
Randomizer

An ensemble alone is not enough.

Mere evidential neutrality over the ensemble members does not induce an additive measure.

Some further element of the evidence must introduce a complementary favoring-disfavoring.

Probabilities from Multiverses?



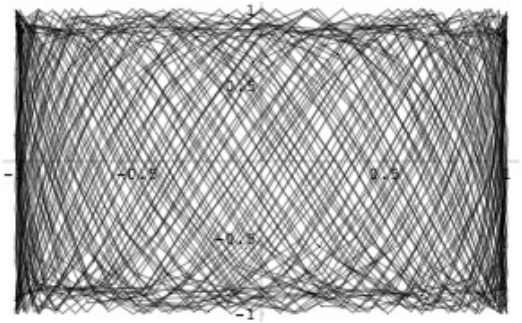
1. A bunch of orbits B cut transversely by hypersurfaces Σ and Σ′.

Gibbons, Hawking, Stewart (1987):
Hamiltonian formulation of general relativity.
Additive measure over different cosmologies induced by canonical measure.

Gibbons, G. W.; Hawkings, S. W. and Stewart, J. M. (1987) "A Natural Measure on the Set of All Universes," Nuclear Physics, B281, pp. 736-51.

Just like the microcanonical distribution of ordinary statistical mechanics?

No: there is no ergodic like behavior and hence no analog of the randomizer.



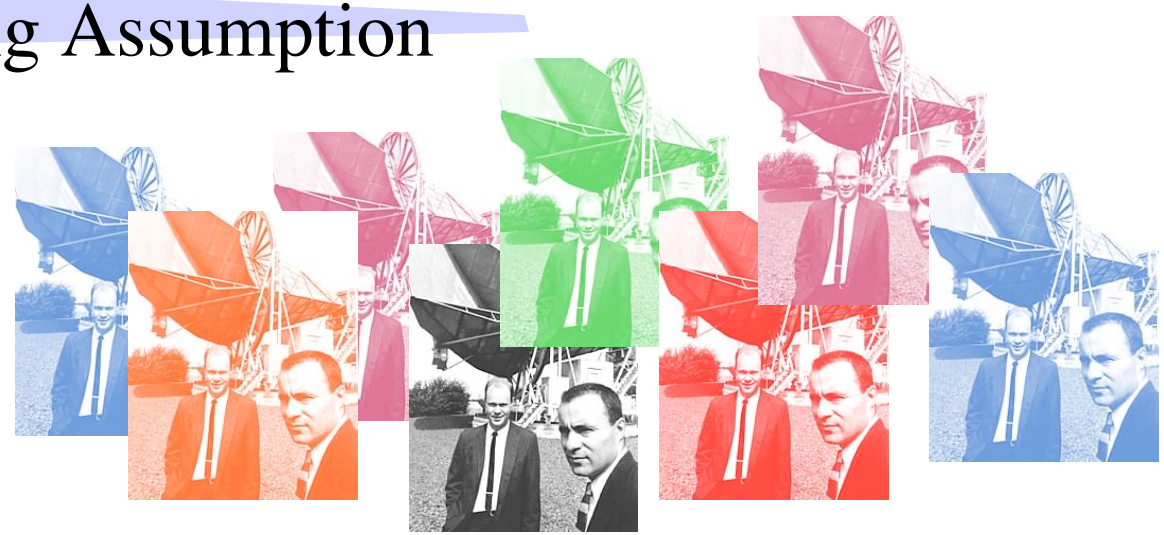
"Giving the models equal weight corresponds to adopting Laplace's 'principle of indifference', which claims that in the absence of any further information, all outcomes are equally likely."

Gibbons, Hawking, Stewart, p. 736

Ensemble without randomizer

The Self-Sampling Assumption

Penzias and Wilson measure 3°K cosmic background radiation.



Level I multiverses. Many clones of Penzias and Wilson measure 3°K cosmic background radiation in other parts of space.

Which is our Penzias and Wilson?

Self-Sampling Assumption: "One should reason if as one were a random sample from the set of of all observers in one's reference class." (Bostrom, 2007, p. 433)

Evidence on which is our PW is neutral. No warrant for a probability measure.

The self-sampling assumption imposes probabilities where they do not belong by mere supposition.

Why have the Self-Sampling Assumption?

$$P\left(\begin{matrix}\text{measure}\\ 3^oK\end{matrix} \,\middle|\, \begin{matrix}\text{back}\\ \text{-ground is}\\ 100^oK\end{matrix}\right) = q << 1$$

"(L)" A physical chance computed in a physical theory.

$$P\left(\begin{matrix}\text{someone}\\ \text{somewhere}\\ \text{measures}\\ 3^oK\end{matrix} \,\middle|\, \begin{matrix}\text{back}\\ \text{-ground is}\\ 100^oK\end{matrix}\right) \text{ is (near) one.}$$

Very many trials carried out in the multiverse.

Introduce self-sampling to reduce this probability by allowing that our PW is probably not the "someone somwhere."

$$P\left(\begin{matrix}\text{Our PW}\\ \text{measure}\\ 3^oK\end{matrix} \,\middle|\, \begin{matrix}\text{back}\\ \text{-ground is}\\ 100^oK\end{matrix}\right) = \sum_i \underbrace{P\left(\begin{matrix}\text{i-th PW}\\ \text{measure}\\ 3^oK\end{matrix} \,\middle|\, \begin{matrix}\text{back}\\ \text{-ground is}\\ 100^oK\end{matrix}\right)}_{q} \underbrace{P\left(\begin{matrix}\text{i-th PW}\\ \text{is our PW}\end{matrix}\right)}_{1/n} = q$$

Recover the same result without sampling or calculation just by applying (L) directly to case of "our PW."

If n = infinity, the computation fails.
"1/n = 1/infinity = 0"
The failure is an artifact of the probabilistic representation and its difficulties with infinitely many cases.

Heretical Thought

Does Everything Require an Explanation?

Surprising Analysis: Certain cosmic parameters are surprising and demand explanation.

Our background is neutral to cosmic parameters. ✗ They are "improbable."

Cosmic parameters are used in the analysis as the basis for an inductive inference to new theory. But sometimes--when background facts are inhospitable--no inductive inference is possible.

Examples: spacetime is inextendible, choosing between observationally equivalent spacetimes.

An infinite regress?

What explains the explainers; and explains the explainers of the explainers….

Winding Up

This Talk

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Fragments of inductive logics that tolerate neutral support displayed.

Non-probabilistic state of *completely* neutral support.

Artifacts are introduced by the use of the wrong inductive logic.

Doomsday argument.

Probabilities are recoverable from multiverses if there is a randomizer.

Self-sampling Assumption.

Inductive inference has no magical Powers

Inductive inference can extend our knowledge beyond the evidence.

The weaker the evidence. → The more fragile the knowledge.
The greater the extension. → The greater the inductive risk.

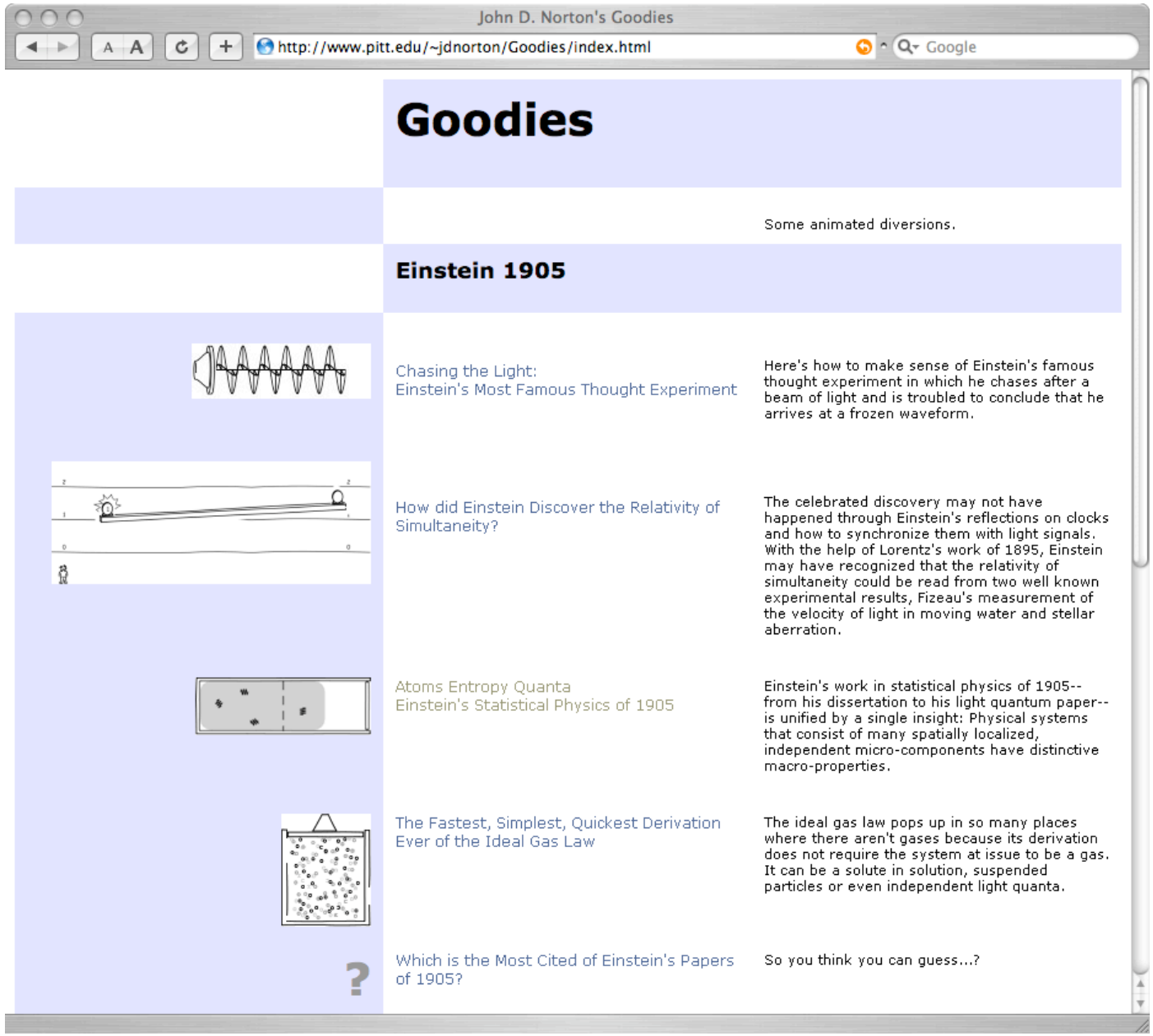
The weakest case is neutral evidence. Expect the least.

Harry Houdini
magician and skeptic

Be skeptical of a system of inductive logic that can still deliver non-trivial results from neutral evidence.

Nihil ex nihil fit.

Read all about it…



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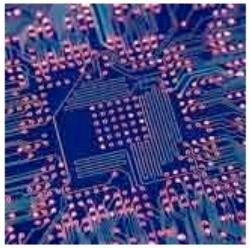

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